The Economics of Skyscrapers: A Synthesis*

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Abstract

We document that skyscraper growth since the end of the 19^{th} century has been driven by a reduction in the cost of height, increasing urbanization, and rising incomes. These stylized facts guide us in developing a competitive open-city general equilibrium model of vertical and horizontal city structure. We use the model to show that (i) vertical costs and benefits affect the horizontal land use pattern within cities; (ii) the causal relationship between skyscrapers and urbanization is bi-directional; and (iii) height limits reduce the size of large cities, leading to lower exposure to agglomeration, productivity, and urban GDP. We substantiate the model's predictions by novel estimates of urban height gradients.

Key words: Density, economics, history, height regulations, skyscraper, urbanization

JEL: R3, N9

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1 Introduction

The study of the *horizontal* structure of urban space has been one of the main research areas in urban economics ever since the birth of the field (Alonso, 1964; Mills, 1967; Muth, 1969). While iconic skylines have long become distinctive features of cities over the course of the 20$^{\text{th}}$ century, it is not until recently that urban economics research into the *vertical* structure of cities has gained momentum.

We bring together recent research on costs (Ahlfeldt and McMillen, 2018; Barr, 2010) and benefits (Liu et al., 2018, 2020; Barr, 2016) of building height, which we integrate into a general equilibrium model of the vertical and horizontal structure of an open city. We use the model to engage with the following questions concerning the positive and normative economics of skyscrapers: How do vertical costs and benefits shape the internal structure of cities? How does the distribution of economic activity between cities and rural hinterlands affect the vertical size of cities and *vice versa*? What are the consequences of distortionary policies that constrain the vertical size of cities on the spatial distribution of economic activity and welfare?

We complement our theoretical analyses with novel evidence on urban height gradients to guide our modeling choices and confirm the model's predictions. From our theoretical and empirical analyses, as well as our reading of the literature, we conclude that vertical costs and benefits, horizontal land use patterns, urban growth, and welfare are all mutually dependent in ways that leave plenty of room for future research.

In developing a model that integrates vertical and horizontal spatial structure, we must engage with the obvious question of how skyscrapers emerge.[1] Therefore, we start by exploiting a unique data set that blends building-level data from commercial and non-profit providers covering the entire planet and hand-collected data for selected cities spanning 150 years (see Appendix Section I for details) to produce a battery of stylized facts that speak to the determinants of building height.

We show that since 1900, there were waves of skyscraper development *when* there were innovations in construction technology and *where* there was economic growth. With few exceptions, the tallest skyscrapers today are not outliers within the skylines of their host cities. Even the Empire State Building, which ruled the height ranking for longer than any other building, delivered a decent return on investment over its lifetime. Our conclusion is that the canonical approach that assumes profit-maximizing developers in a competitive market provides a reasonable description of height decisions, even for most tall buildings.

With this in mind, we have developed a new general equilibrium model of vertical and horizontal city structure to study the causes and effects of vertical growth. To engage with our main research questions, we design the model to serve three purposes. First, the

[1] Canonical urban models assume profit-maximizing developers in a competitive market (Brueckner, 1987; Duranton and Puga, 2015; Ahlfeldt et al., 2015). However, if developers compete to win the prize of being the tallest, heights may not be justifiable by economic fundamentals (Helsley and Strange, 2008; Barr, 2010, 2013; Barr and Luo, 2020).

model should rationalize the remarkable differences in building heights *within* cities that are characteristic of metropolitan skylines and account for how use-specific vertical costs and benefits affect the horizontal land use pattern. Second, the model should address the mechanisms that drive the close relationship between urbanization, vertical growth, and rising incomes *between cities*, which we document in our data. Third, the model should be suitable for the evaluation of a common form of land use regulation that constraints vertical space: building height limits. Here, we are interested in the effects that this distortionary policy has on the spatial allocation of economic activity *within* and *between* cities.

Our basic set up is an open-city model (Roback, 1982) with endogenous land use (Duranton and Puga, 2015) and returns to agglomeration in the form of higher productivity. We borrow the supply side of land markets from Ahlfeldt and McMillen (2018), i.e., we impose that the marginal cost of supplying one unit of floor space increases in height at a use-specific elasticity (Barr, 2010). Thus, the model features a labor market related agglomeration force in the form of higher wages and a housing market related dispersion force in the form of higher rents.[2] As for the geography, we consider a linear city where residential and production amenities differ across both horizontal and vertical space. We summarize accessibility benefits by a monotonic function of distance from a historic core and allow for greater production and residential amenity values at higher floors to rationalize recent evidence on positive vertical rent gradients (Liu et al., 2018; Danton and Himbert, 2018).

Perfect competition among firms and perfect mobility of residents leads to perfect spatial arbitrage so that differences in amenities map into differences in rents in horizontal and vertical space. Profit-maximizing developers respond to higher rents by building taller. Perfect competition results in full capitalization of developer profits in land rents. Land is allocated to the use that generates the highest land rent, pinning down the endogenous size of the central business district (CBD) and the residential zone. In equilibrium, the city-specific wage and population, as well as location-specific floor space rents adjust such that labor and land markets clear and the utility within the city equates to the exogenous reservation utility. In this setting, an exogenous change in any parameter will generally lead to adjustments in all endogenous outcomes. Hence, the model is well suited to account for the mutual dependence of use-specific *vertical* costs and benefits, *horizontal* patterns of land use, and the size and productivity of a city.

Before we can use the model to quantitatively evaluate these mutual dependencies, we need to settle on canonical values for the model's structural parameters. Two of them deserve particular empirical attention because evidence is scarce: The height elasticity of per-unit construction cost and the height elasticity of per-unit floor space rent, which impact the costs of and returns to height. For the former, we estimate a range from 0.1 for smaller structures to 0.25 for tall commercial and 0.56 for tall residential structures.

[2] See for a summary of the related empirical evidence on the economic effects of density, see Ahlfeldt and Pietrostefani (2019).

For the latter, we estimate a value of about 0.07 for residential units in New York City, as well as in Chicago. For New York City, we estimate a value of 0.033 for commercial rents. These estimates reveal that the costs of, as well as the returns to, height are greater for residential than for commercial land use.[3]

The importance of accounting for such use-specific differences to understand the urban height gradient becomes apparent in the first application of our model. Under the baseline parametrization, the model delivers canonical height and rent gradients that are downward-sloping in distance from the city center and a central business district (CBD) surrounded by a residential zone (Duranton and Puga, 2015). The lower net-cost of height for commercial buildings results in steeper commercial height and land rent gradients. Importantly, we obtain the novel prediction that the commercial and residential land bid rents intersect nearer to the center than the floor space rent gradients. This pushes the CBD boundaries inwards and generates discontinuities in the height and the floor space gradients at the edge of the CBD. Hence, our model rationalizes why tall CBDs often stand out relative to the surrounding residential zones through endogenous mechanisms and without requiring exogenous zoning or spatial discontinuities in the distribution of amenities.

The model predictions for the height gradient are consistent with patterns that we find in the data. We estimate an elasticity of building height with respect to distance from a "prime location" (business centers identified by Ahlfeldt et al., 2020) of about -0.25 for a global set of cities in 1900 and 2015. It appears that demand-side forces pushing for a flatter height gradient and supply-side forces pushing for a steeper height gradient have largely offset each other in their effect on the slope of the urban height gradient over the 20^{th} century.

Our estimates confirm that commercial rent gradients are steeper than their residential counterparts. We also find evidence for a discontinuity in the height gradient at the edge of CBDs. Conditional on smooth gradients inside and outside the boundary, there is a jump in building heights of slightly more than 20% as one steps into the CBDs of the U.S. cities in our data. Finally, we discretize space and invert the distributions of residential and commercial amenity under which the model matches the land use pattern and the skyline of Chicago. We find that small differences in the amenity value are able to rationalize a fuzzy height gradient with large differences in building heights. Under the inverted amenity values, the model generates a distribution of land values that closely resembles observed data, lending support to the parametrization of our model.

In the second application of the model, we turn to the obvious stylized fact that skyscrapers are a big-city phenomenon. A standard prediction of urban land use models is that, because the horizontal expansion is constrained by commuting costs, a larger city size maps into greater structural density, i.e., taller buildings (Brueckner, 1987; Duranton and Puga, 2015). A separate literature advocates the reverse causality by arguing that a favorable geography (solid bedrock near the surface) facilitates the construction of

[3]We also find that costs and benefits likely decreased over time.

tall buildings, leading to agglomeration and greater productivity (Rosenthal and Strange, 2008; Combes et al., 2011). In a series of comparative statics that engage with major long-run trends observed over the 20th century, we illustrate how the causal relationship between building height and city population is, indeed, bi-directional. A rise in returns to agglomeration increases the wage and attracts workers to large cities, causing skyscrapers to emerge via a demand-side channel. Similarly, a reduction in the horizontal cost of travel not only leads to a horizontal expansion, but also to a vertical expansion, since the heightened attractiveness of the city results in greater demand for floor space.

In contrast, a reduction in the cost of height leads to taller buildings via a supply-side channel and lower floor space rents, which causes the city to grow until the equilibrium is restored. Hence, the sizable reduction in the cost of building tall could be an important driving force of urbanization that has gone unnoticed in a literature concerned with the growth of cities.[4]

In the third application of the model, we evaluate how the spatial structure of cities responds to height limits. In keeping with intuition, a vertical compression leads to a horizontal expansion of the CBD. The effect on the residential zone, however, is the opposite. In our open-city model, the supply-driven increase in floor space rents leads to out-migration until an equilibrium is restored at a smaller population living on less horizontal space. The wage level falls owing to smaller agglomeration economies and, eventually, rents fall below the free-market equilibrium to restore the reservation utility. Our model predictions substantiate a popular notion in the literature that height regulation can have large and negative welfare consequences (Glaeser et al., 2005); yet, they also reveal that, if workers are mobile, relaxing height restrictions in the most expensive cities may have smaller impacts on affordability than advocates may wish for (Glaeser et al., 2005), at least in the long-run.[5] In any case, a complete welfare appraisal of height regulations will have to account for the reductions in the presumed negative externalities from building height, such as increased shadows, reduced sunlight, increased traffic congestion, or lost historic charm.

The remainder of the paper is organized as follows. Section 2 introduces our data and provides stylized facts that motivate some of the choices that we made when developing our model in Section 3. Section 4 discusses our choices of parameter values. Section 5 compares the model predictions for vertical and horizontal spatial structure with the evidence. Section 6 uses the model to explore the reciprocal relationship between urbanization and vertical growth. Section 7 analyzes the impacts of height caps on welfare and the vertical and horizontal urban structure. Section 8 concludes by summarizing some priority areas for future research. For the interested reader, we complement each section in this paper with a dedicated section in the appendix in which we provide further theoretical and empirical analyses, discuss the related literature in more detail, and suggest directions for

[4]See, for example, Glaeser et al. (1992); Henderson et al. (1995); Duranton and Turner (2012).

[5]But it would generate sizable economic benefits, such as higher average productivity due to greater exposure to external returns to scale (Hsieh and Moretti, 2019).

future research.

2 Data sets and stylized facts

Before we develop our model of vertical and horizontal structure in Section 3, it is useful to review some stylized facts that suggest that a conventional competitive model with an emphasis on profit-maximization is a reasonable, if not perfect, description of height decisions. We find that vertical growth appears to be a rational response to changing demand and supply conditions. Suspiciously-tall buildings, whose height is likely determined by non-economic motives, appear to be the exception rather than the rule. We introduce the data set we use in this and the subsequent sections in Section 2.1 before we turn our attention to the spatiotemporal diffusion of skyscrapers in Section 2.2 and the particularities of super tall buildings in Section 2.3.

2.1 Data

For our empirical work, we combine data that fall into four categories, which we briefly discuss below. More information on sources can be found in Appendix Section I. First, we exploit geo-coded data sets containing building heights, use, completion dates, and construction costs of tall buildings all over the world that are available from commercial and non-for-profit data providers, such as Emporis and the CTBUH Skyscraper Center. We complement these data with administrative data on all buildings for New York and Chicago obtained from the respective authorities.

Second, we use commercial rent and residential real estate prices with a geo-coded building reference for New York and Chicago from Cushman & Wakefield, Streateasy.com, Redfin.com, as well as geo-coded commercial rents for a wider set of global cities from SNL-S&P. For geo-coded land prices in Chicago and New York spanning more than a century, we rely on land values from Hoyt (1933), *Olcott's Land Values Blue Books of Chicago*, as well as vacant land sales from Ahlfeldt and McMillen (2018), Barr et al. (2018), Fred Smith (Davidson College), and Spengler (1930).

Third, we look at hand-collected data sets. We obtain gross and net floor areas for a global sample of buildings from Sev and Ozgen (2009), Watts et al. (2007), Kim (2004), and Berger (1967). We collected cash flows for the Empire State Building throughout the 20^{th} century from files located in the Hagley Museum Archives, newspaper articles, and SEC 10-K forms.

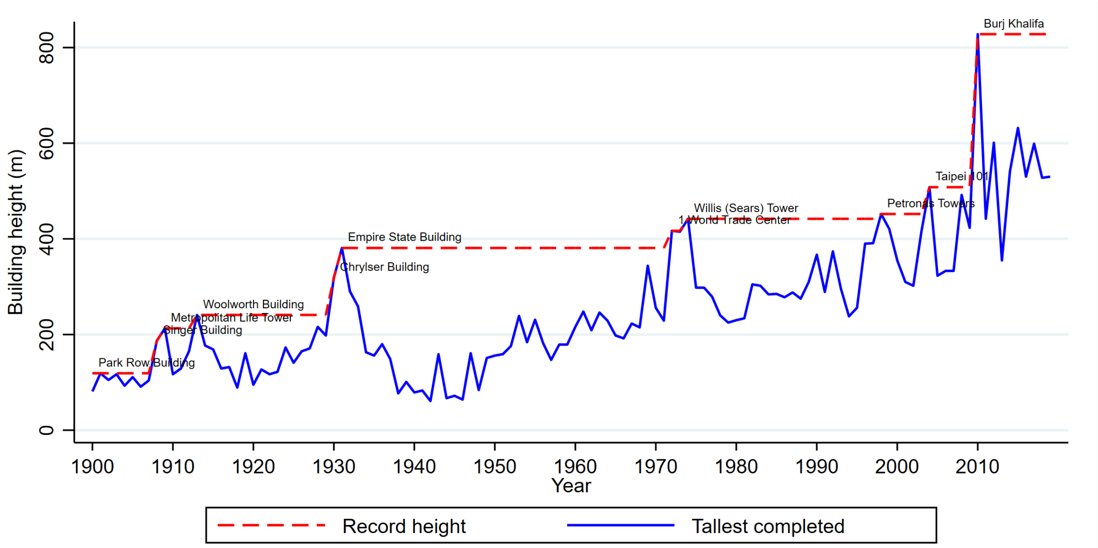
Fourth, we analyze global cross-sections of population, GDP and national macroeconomic time-series that are more readily accessible from institutions, such as the World Bank or MeasuringWorth.com.

2.2 Spatiotemporal patterns in skyscraper diffusion

Figure 1 illustrates the height of the tallest skyscraper completed each year over the 20th and 21st centuries.[6] The pattern is cyclical, and there are several peaks throughout history. From 1908 to 1913, three buildings held the title of the tallest building in the world, all exceeding 200 meters and all located in New York City. World War I induced a period of less ambitious construction before the next wave of skyscraper development culminated in the famous skyscraper race in which the Chrysler Building was defeated by the iconic 380-meter-tall Empire State Building.

Following the Great Depression and World War II, it took until the 1950s before the tallest buildings started to reach the levels of the late 1920s. A new benchmark was reached in the 1970s when the Twin Towers took the crown from the Empire State Building after nearly four decades. It was rapidly overtaken by the 440-meter-tall Willis (Sears) Tower in Chicago, the first record-breaking skyscraper outside of New York City. Following the oil crisis, there was a dip in tallest buildings until the 1990s when skyscraper development gained new momentum, this time in a much more international context. The Petronas Towers (1998) and the Taipei 101 (2004) pushed the limits to 452 and 508 meters, respectively, before the Burj Khalifa set the record of 828 meters in 2009. Since then, it has become the norm that the tallest completed structures within a year exceed 500 meters (about 100 floors).

Figure 1: Tallest completions



Notes: Sources: Data from https://www.emporis.com/ and https://www.skyscrapercenter.com/

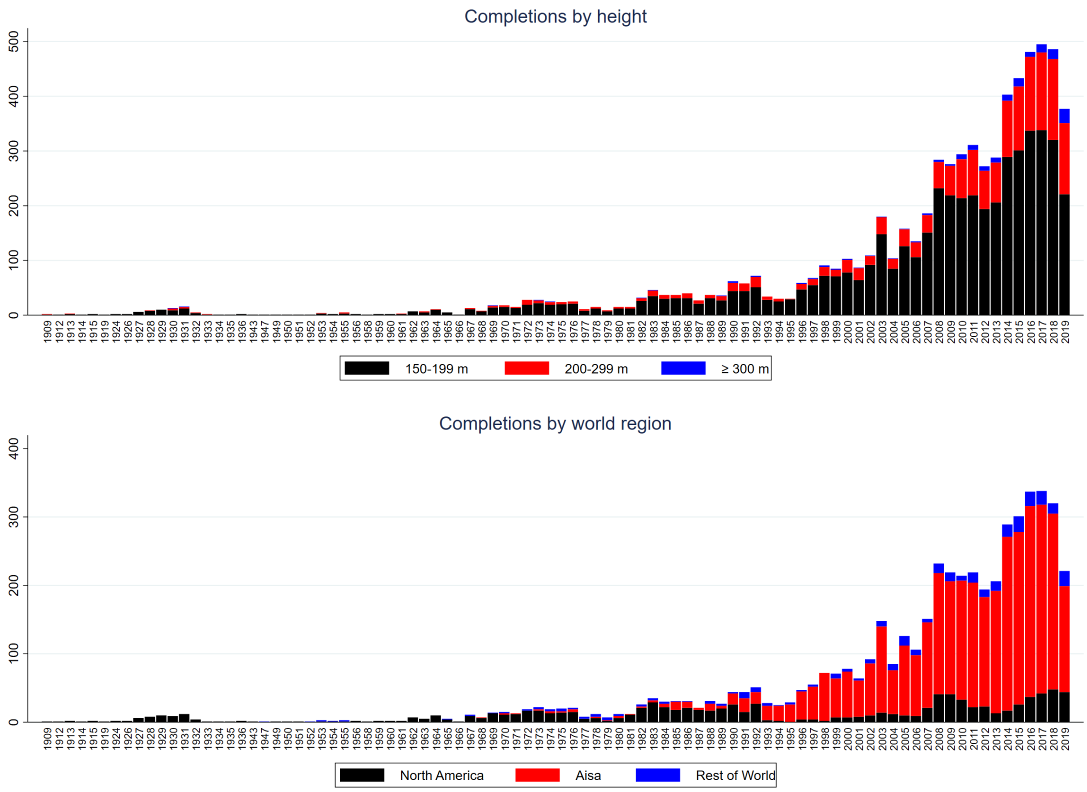
These waves of development during the late 19th century, the 1970s, and the turn of the 19th to the 20th century coincide with both periods of economic growth in North America

[6]For simplicity, we define a "skyscraper" as an occupiable structure that is at least 150 meters tall.

and Asia and seminal innovations in construction technology.[7] Early skyscrapers were facilitated by the steel-framed skeletal structure and the electric elevator that become available around 1900. From the 1960s onward, mainframe computing allowed for the implementation of new structural techniques that require less steel per cubic meter, such as the framed-tube structure. Further refinements at the beginning of the 21st century, such as the buttressed core in the Burj Khalifa, coincide with yet another jump in the height of the tallest completions (see Appendix Section J.1 for a review of the technological history of skyscrapers).

The same cyclically is visible in the number of completed skyscrapers, depicted in Figure 2. Driven by Asia, the pace of vertical growth has accelerated since the 1990s. In 2002, Asia took over from North America as the region with the largest cumulative number of skyscrapers. Today, 43% of the world's 150-meter or taller towers are in China alone (including Hong Kong).

Figure 2: Skyscraper completions by year



Note: A skyscraper is a building that is at least 150 m tall. Source: https://www.skyscrapercenter.com/

To quantify the positive long-run trends in heights and completion counts of 150-meter or taller buildings, it is useful to regress the log of heights and completions against a yearly time trend. Over 120 years, the heights of the tallest building have increased at an

[7] Barr et al. (2015) finds that GDP Granger-causes height, but that height does not predict GDP.

average rate of 1.3%. The completions count has increased at an even larger percentage of 4.9%. Given that simple log-linear trends explain 63% and 82% of the variation in tallest heights and skyscraper counts over time, it seems fair to conclude that pronounced vertical growth, historically, has been the norm rather than the exception (see Table A1 in Appendix Section J).

In Figure 3, we take a closer look at the spatial diffusion of skyscrapers. The first skyscraper outside the U.S. was completed in Brazil (the Altino Arantes Building) in the 1940s. Skyscrapers reached many of the larger and economically more developed countries during the second half of the 20$^{\text{th}}$ century. Consistent with a shift in gravity from the developed to the developing world, economic growth has become the primary determinant of vertical growth, in addition to the level of GDP (see Figure A1 Appendix J).

Still, only 59 out of 193 nations (30%) have at least one skyscraper.[8] Some developed countries have restricted the adoption of the technology by means of land use regulation. For example, it took until the 2010s for skyscrapers to reach Italy and Switzerland. Ireland and Portugal do not have any skyscrapers (150 meters or taller) to date.

The degree of urban bias in the distribution of skyscrapers is striking. Small states in which the largest city dominates the city system, such as Panama or the United Arab Emirates, reach the highest penetration in per capita terms. 50% of the world's skyscrapers are located in just 17 cities, though a total of 315 cities worldwide have at least one. In terms of 150-meter or taller skyscrapers, Hong Kong leads the list with 353, with New York City coming in second at 281. Of the top twenty cities around the world, nine are in China (including Hong Kong). 16 out of 20 are in east Asia. Only three cities in North America (New York, Chicago, and Toronto) make the list.[9]

To summarize, it seems reasonable to conclude that skyscrapers tend to break height records when new technologies become available and where urbanization and economic growth create the necessary demand, unless regulation prevents the adoption of the technology.

2.3 "Too tall" buildings

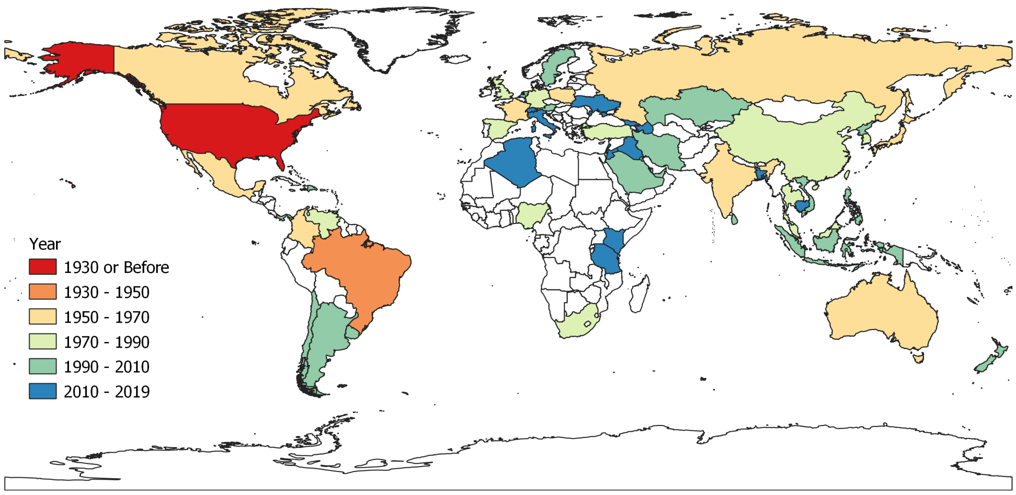
While the spatiotemporal diffusion of skyscrapers appears to reflect the interplay of demand and supply conditions, there is nevertheless a common belief that the height of the tallest buildings is driven by non-pecuniary motivations. As an example, the "height race" in New York City during the 1920s led to the perception that developers of tall buildings, in aiming to dominate the skyline, seek to satisfy their egos rather than maximize profits (Barr, 2016).[10] Indeed, Helsley and Strange (2008) offer a game-theoretic model that rationalizes how developers vying to claim the prize of the "tallest building" build "too tall" to preempt rivals (see Appendix Section J.6 for a discussion of the related literature).

[8] 80% of all skyscrapers are in eight countries.

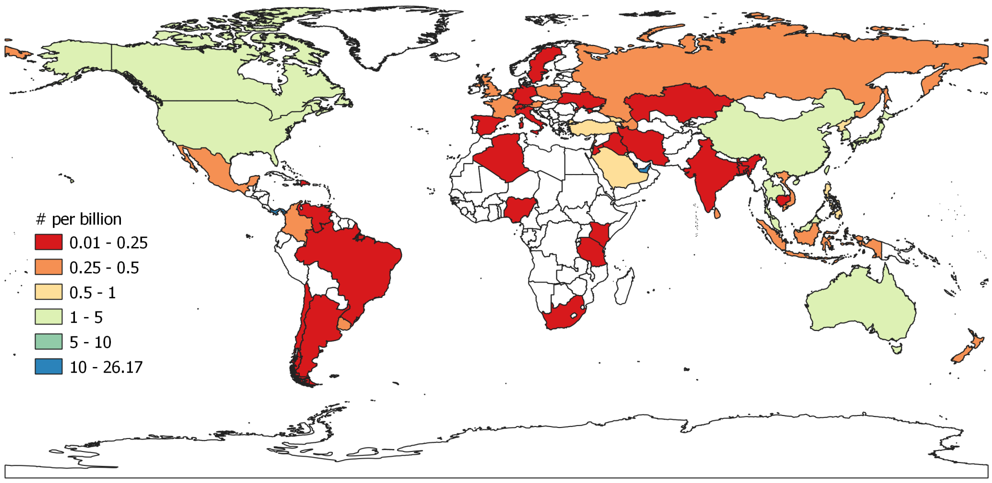
[9] See https://www.skyscrapercenter.com/, accessed May 1, 2020.

[10] Bercea et al. (2005) argue that in the 16$^{\text{th}}$ century, the Roman Catholic church used cathedrals to signal their power when faced with competition from Protestantism.

Figure 3: Skyscraperization by country



(a) Year of first skyscraper



(b) Skyscrapers per capita

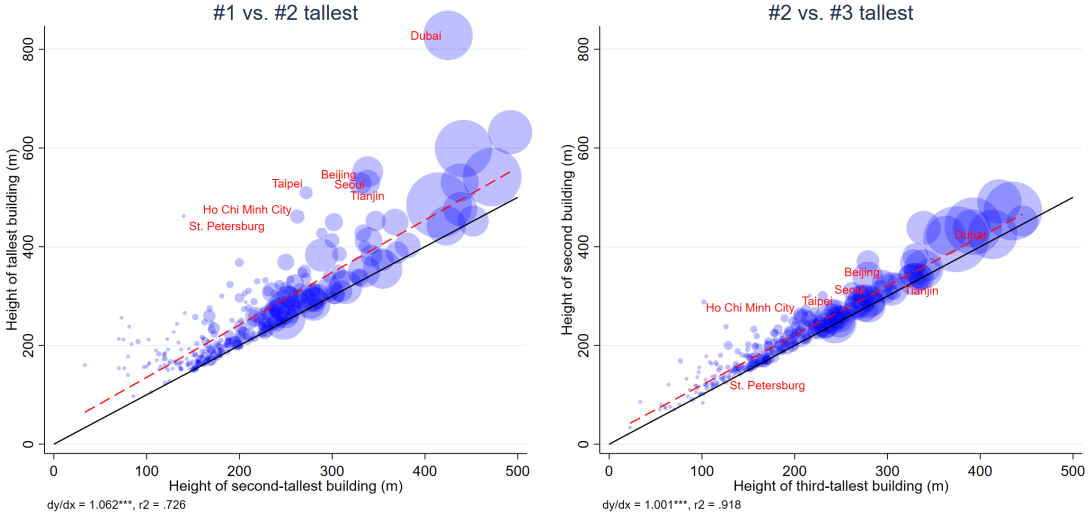
Note: A skyscraper is a building that is at least 150 meters tall. Source: https://www.skyscrapercenter.com/, as of 2018. Population data from https://data.worldbank.org/.

It is difficult to judge objectively by how much the height of an iconic building exceeds the fundamentally justified height. However, it is reasonable to expect that if pecuniary motives dominate, the second-tallest building in a city will be a good predictor of the height of the tallest building in a city. Indeed, we find a strong correlation across a large set of cities that adopted the skyscraper technology, as shown in Figure 4, but there are some outliers, which include obvious candidates such as Dubai, home to the Burj Khalifa, the tallest building on the planet. If we turn to the heights of the second and third tallest buildings, however, the correlation is strikingly close.[11] Hence, it seems likely that

[11]The only notable outlier is Ho Chi Minh City in Vietnam, where skyscrapers are a recent phenomenon.

economic fundamentals drive the heights of nearly all but a handful of the tallest buildings in a limited number of cities. Consistent with this interpretation, we find that only few buildings managed to hold the crown of being the tallest for a very long time. While the Empire State Building ruled height rankings for 40 years, the median duration is six years at the global and four years at the national scale (see Appendix Section J.3). This relatively rapid succession is consistent with a steady rise of the profit-maximizing building height, due to increasing demand and improving construction technology.

Figure 4: Tallest building heights



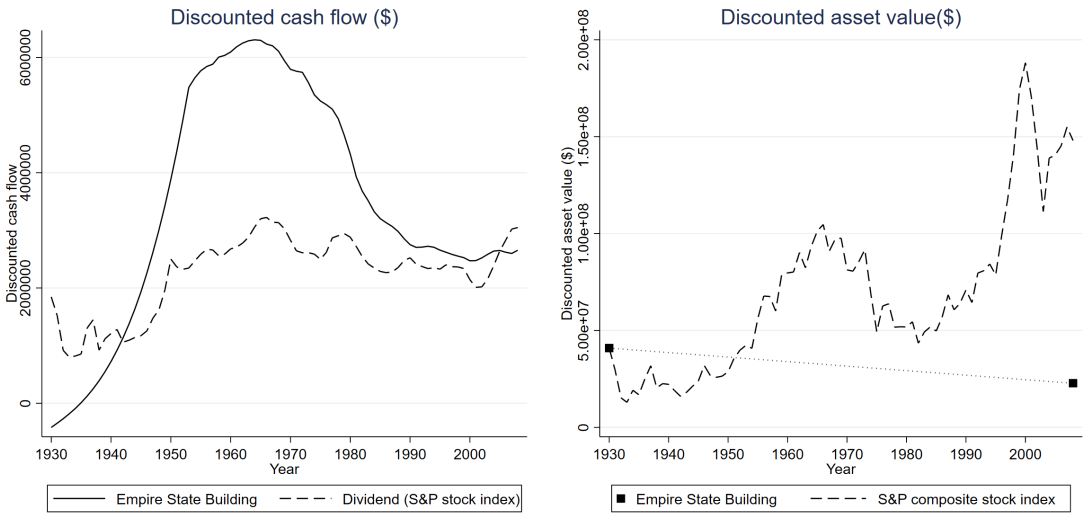
Note: Data covers 315 cities around the world with at least one skyscraper (heights $\geq$ 150 meters. Marker size is proportionate to the number of skyscrapers in a city. Data are from Emporis (accessed in April, 2021).

Closely related to the presumption that tall buildings serve non-pecuniary motives is the notion that they are not particularly profitable. For example, in 1930, Clark and Kingston (1930) wrote their book, *The Skyscraper: Study in the Economic Height of Modern Office Buildings*, to argue that those who called skyscrapers “freak buildings” misunderstood the economics of building tall. They argued that tall buildings were inherently a function of high land values, and they aimed to silence critics who felt that tall buildings were an inefficient use of land.

Critics, however, have not been convinced. The Empire State Building (ESB), in particular, became infamous as a stereotype of a loss-making “too tall” building (Kingwell, 2006). To move beyond anecdotes that mainly refer to high vacancy in the ESB during 1930s, we collected archival records and recent annual reports up until 2009, when a half billion dollar renovation marked the beginning of a new life for the ESB. Figure 5 presents the key results of an ex-post financial case study whose details we report in Appendix Section J.4. Indeed, the ESB took a greater hit than the stock market during the Great Depression. Yet, the recovery was also stronger. During the 1950s, 60s, and 70s, the ESB’s net operating income exceeds the dividend of a stock market portfolio with the same initial asset value as the ESB by a large margin.[12]

In terms of asset value, the stock market portfolio naturally outperforms the ESB as its structure capital, which accounts for about two thirds of the initial asset value, depreciates to zero. However, over the (first) life-cycle of the ESB, the higher discounted net operating income weighs more (as it occurs earlier). As a result, the ESB delivered a rate of return on investment of 5.4% on top of the risk-free rate, beating the stock market, which achieves 4.3%. Adjusting for the adverse economic conditions during the 1930s and 1940s, the return would have exceeded 8%, which was a good return for 1929 Manhattan (Clark and Kingston, 1930). The useful lesson for our model in the next section is that even the developers of the world's arguably most iconic skyscraper were not willing to compromise much on their ambition to make profits.

Figure 5: Empire State Building vs. stock market



Note: ESB asset value is total cost of structure and land in 1930 and land value in 2008 (as we assume that the structure has fully depreciated). For stock market, the asset value is the total investment inflated by the stock market index. Cash flow of the ESB is the net operating income (EBITDA). Cash flow of the stock market is the dividend (the product of the dividend yield and the asset value). All time series deflated by the cummulated opportunity cost of capital (risk-free rate). The net realised return for the ESB, at 5.4%, beats the stock market, at 4.3%. See Appendix Section J.4 for details.

3 Model

Guided by the stylized facts introduced in Section 2, we now develop a general equilibrium model of urban land use. The model features various canonical elements of standard land use models reviewed by Duranton and Puga (2015). The novel element is that we integrate use-specific vertical costs and benefits into an open-city model in which wages, population,

[12]During this period, the ESB was purchased by Harry Helmsley (in 1961) who built a reputation for developing a highly profitable portfolio that made him a real estate billionaire.

and land use are endogenously determined. Our aim is to demonstrate how the interplay of demand for space and use-specific net-costs of height shape skylines and horizontal land use patterns. Hence, we keep the household and firm location problem as simple as possible.[13] We provide a fuller discussion of the related theoretical literature, as well as potential for future research in Appendix Section K. Since we will use the model for quantitative evaluations throughout Sections 5 to 7, we develop the model using fairly restrictive, yet canonical functional forms.

Geography. We consider a linear city of endogenous length. One unit of land is available at each location, x, for development. The point $x = 0$ marks the historic center of the symmetric city, with $D(x) = |x|$ being the distance from the center.

Workers. Identical workers earn a city-specific wage, y. A worker living at floor s of a building located at x derives a Cobb-Douglas utility from the consumption of a tradable good, g, whose price is normalized to one and residential floor space f^R:

$$U(x, s) = A^R(x, s)\left(\frac{g}{\alpha^R}\right)^{\alpha^R}\left(\frac{f^R(x, s)}{1-\alpha^R}\right)^{1-\alpha^R}, \tag{1}$$

where $0 < (1 - \alpha^R) < 1$ is the expenditure share on floor space. Utility depends on the location in vertical (s) and horizontal (x) space via $A^R(x, s) = \tilde{A}^R(x)s^{\tilde{\omega}^R}$, where $\tilde{A}^R(x) = \bar{a}^R e^{-\tau^R|x|}$. $\tilde{\omega}^R > 0$ is the height elasticity of residential amenity that captures height benefits, such as better views or less exposure to noise and pollution. $\tau^R > 0$ is a decay parameter that determines the rate at which utility declines in distance from the historic center and $\bar{a}^R$ moderates the utility at the center. Travel is inconvenient but free; hence, consumption is subject to the budget constraint $y = p^R(x, s)f^R(x, s) + g$, where $p^R(x, s)$ is the residential floor space rent.[14] One way to rationalize this setting is to assume that residents make a daily shopping trip by public transit to the historic center to purchase the tradable good and stop along the way for work. Utility maximization delivers the Marshallian demand functions $g = \alpha^R y^R$ and $f^R(x, s) = \frac{1-\alpha^R}{p^R(x,s)^{1-\alpha^R}} y^{\alpha^R}$ and the indirect utility $U(x, s) = A^R(x, s)\frac{y^R}{p^R(x,s)^{1-\alpha^R}}$. Assuming that residents are perfectly mobile within and across cities, utility is anchored to the reservation utility, which we normalize to $\bar{U} = 1$ without loss of generality. Setting $U(x, s) = \bar{U}$, we can solve for the residential bid rent, $p^R(x, s) = A^R(x, s)^{\frac{1}{1-\alpha^R}} (y^R)^{\frac{1}{1-\alpha^R}}$. Averaging across all floors of a

[13] We refer the interested reader to the literature for a more realistic treatment of commuting (see e.g., Fujita and Ogawa, 1982; Lucas and Rossi-Hansberg, 2002; Ahlfeldt et al., 2015)

[14] This formulation is a hybrid between quantitative spatial models in which commuting costs are accounted for by a utility shift that exponentially depends on distance from the actual workplace location (Ahlfeldt et al., 2015) and canonical monocentric city models in which commuting costs increase linearly in distance from the CBD and affect utility indirectly via the budget constrain (Brueckner, 1987). Our choice is convenient because we obtain the same functional forms for the commercial and residential bid rents.

building of height $S^R(x)$ at any location, x, we obtain the horizontal residential bid rent:

$$\bar{p}^R(x) = \frac{1}{S^R(x)} \int_0^{S^R} p^R(x,s)ds = \frac{a^R(x)}{1+\omega^R} S^R(x)^{\omega^R}, \tag{2}$$

where $a^R(x) = \tilde{A}^R(x)^{\frac{1}{1-\alpha^R}} (y^R)^{\frac{1}{1-\alpha^R}}$ and $\omega = \frac{\tilde{\omega}}{1-\alpha^R}$ is the height elasticity of residential rent. The average floor space demand at x is then given by $\bar{f}^R(x) = \frac{1-\alpha^R}{\bar{p}^R(x)} y^R$.

Firms. Atomistic firms produce the tradable good using labor, l, and commercial floor space, f^C, in a Cobb-Douglas production function:

$$g(x,s) = A^C(x,s) \left(\frac{l}{\alpha^C}\right)^{\alpha^C} \left(\frac{f^C(x,s)}{1-\alpha^C}\right)^{1-\alpha^C}, \tag{3}$$

where $0 < (1-\alpha^C) < 1$ is the factor share of floor space. Productivity is shifted in vertical (s) and horizontal (x) space by $A^C(x,s) = \tilde{A}^C(x) s^{\tilde{\omega}^C}$, where $\tilde{A}^C = \bar{a}^C N^\beta e^{-\tau^C |x|}$. $\tilde{\omega}^C > 0$ is the height elasticity of production amenity that captures height benefits, such as signaling and workplace amenity effects (Liu et al., 2018). The agglomeration elasticity of productivity $\beta > 0$ determines how productivity increases in city employment N (Combes and Gobillon, 2015). $\tau^R > 0$ is a decay parameter that determines the rate at which productivity declines in distance from the historic center and $\bar{a}^R$ moderates the productivity at the center. One way to rationalize this setting is to assume that all workers have to meet at the historic center to exchange knowledge. Maximization of the profit function $\pi(x,s) = g(x,s) - yl - p^C(x,s) f^C(x,s)$ delivers the marginal rate of substitution $\frac{l(x,s)}{f^C(x,s)} = \frac{\alpha^C}{1-\alpha^C} \frac{p^C(x,s)}{y^C}$, which is used in the profit function and assuming perfect competition and zero profits delivers the commercial bid rent $p^C(x,s) = A^C(x,s)^{\frac{1}{1-\alpha^C}} (y^C)^{\frac{\alpha^C}{\alpha^C - 1}}$. Averaging across all floors of a building with height $S^C(x)$ at any location x, we obtain the horizontal commercial bid rent:

$$\bar{p}^C(x) = \frac{1}{S^C(x)} \int_0^{S^C} p^C(x,s)ds = \frac{a^C(x)}{1+\omega^C} S^C(x)^{\omega^C}, \tag{4}$$

where $a^C(x) = \tilde{A}^C(x)^{\frac{1}{1-\alpha^C}} (y^C)^{\frac{\alpha^C}{\alpha^C - 1}}$ and $\omega^C = \frac{\tilde{\omega}^C}{1-\alpha^C}$ is the height elasticity of commercial rent.

Developers. Each unit of land at x can be occupied by one building that can house one use $U \in \{C, R\}$, where C indexes commercial and R indexes residential use. This assumption is consistent with tall buildings being highly specialized in terms of use (see Appendix Section K.1 for evidence). Perfectly malleable buildings of height, S^U, are constructed by developers facing the profit function:

$$\pi^U(x, S^U(x)) = \bar{p}(x) S^U(x) - \tilde{c}^U(S^U(x)) S^U(x) - r^U(x), \tag{5}$$

where $\tilde{c}^U = c^U S^U(x)^{\theta^U}$. The baseline per-unit construction cost of a one-story building c^U is inflated in height by the height elasticity of construction cost $\theta^U > \omega^U$, which reflects that taller buildings require more sophisticated structural engineering and facilities, such as elevators. r^U is the land rent. Using Eqs. (2) and (4) in Eq. (5), we obtain the profit-maximizing building height $S^{*U}(x) = \left(\frac{a^U}{c^U(1+\theta^U)}\right)^{\frac{1}{\theta^U-\omega^U}}$. The chosen height by the developer is

$$\tilde{S}^U(x) = \min(S^{*U}(x), \bar{S}^U), \tag{6}$$

where $\bar{S}^U$ is a height limit that may be imposed by the planning system. Using $\tilde{S}^U(x)$ in Eq. (5), we obtain the use-specific bid rent for land under perfect competition and zero profits:

$$r^U(x) = \frac{a^U}{1+\omega^U}\left(\tilde{S}^U\right)^{1+\omega^U} - c^U\left(\tilde{S}^U\right)^{1+\theta^U} \tag{7}$$

Equilibrium. Land is allocated to the use associated with the highest land bid rent:

$$S^C(x) = \tilde{S}^C(x), S^R(x) = 0, \text{ if } r^C(x) \geq r^R(x), r^C \geq r^A \tag{8}$$

$$S^R(x) = \tilde{S}^R(x), S^C(x) = 0, \text{ if } r^R(x) > r^C(x), r^R \geq r^A, \tag{9}$$

where r^A is the agricultural rent. Under the restrictions $r^C(x=0) > r^R(x=0)$, $\frac{\partial r^U}{\partial D} < 0$, $\frac{\partial r^C}{\partial D} < \frac{\partial r^R}{\partial D}$ that are consistent with plausible parameter values, there are two points $\{-x_0, x_0\}$ at which the the commercial and residential land rents equate $(r^C(\pm x_0) = r^R(\pm x_0))$. Between these points, commercial developers outbid residential developers when competing for land; thus these points define the boundaries of the central business district (CBD). Similarly, there are two points $\{-x_1, x_1\}$ where the residential and agricultural land rents intersect $(r^R(\pm x_1) = r^A)$ and the city ends. Real estate markets clear, implying that all floor space supplied is input into either consumption or production:

$$F^C(x) = S^C(x), x \in (-x_0, x_0) \tag{10}$$

$$F^R(x) = S^R(x), x \in (-x_1, -x_0) \cup (x_0, x_1), \tag{11}$$

where $F^C(x)$ is the total input of floor space of all firms at x and $F^R(x) = \bar{f}^R(x)n(x)$ is total floor space consumption by all workers $n(x)$ at x. Using Eq. (10), we obtain labor demand at each location via the marginal rate of substitution:

$$L(x) = \frac{\alpha^C}{1-\alpha^C}\frac{\bar{p}^C(x)}{y^C}S^C(x) \tag{12}$$

Using Eq. (10) in the Marshallian demand function, labor supply at any location is given by

$$n(x) = \frac{S^R(x)}{y^R}\frac{\bar{p}^R(x)}{1-\alpha^R}. \tag{13}$$

Finally, aggregate labor market clearing requires that

$$\int_{-x_0}^{x_0} L(x)dx = \int_{-x_1}^{x_0} n(x)dx + \int_{x_0}^{x_1} n(x)dx = N \tag{14}$$

We take parameters $\{\alpha^U, \beta, \omega^U, \theta^U, \tau^U, \bar{a}^U, \tilde{c}^U, \bar{S}^U, \bar{U}, r^a\}$ as given and treat $\{y, N\}$ as city-wide endogenous objects for which we solve, along with the location-specific variables $\{L(x), n(x), \bar{p}^U(x), r^U(x), \tilde{S}^U(x)\}$, using the equilibrium conditions in Eqs. (2), (4), (6), (7), (8), (12), (13), (10), (14) and a straightforward numerical procedure discussed in Appendix Section K.2.

Intuition. The intuition behind our simple land use model is that greater amenity values of locations near the center translate into higher bid-rents and greater profit-maximizing building heights as long as the height elasticity of construction cost exceeds the height elasticity of rent ($\theta^U > \omega^U$).[15] Higher floor space rents and greater building heights lead to larger profits that fully capitalize into higher land rents. Since the general equilibrium of the model is defined by labor and land market clearing and free mobility anchors utility to a reservation level, an exogenous change in any parameter will generally lead to adjustments in all endogenous outcomes. We will explore the land use pattern the model generates in more detail in Section 5 and conduct several comparative static analyses in Sections 6 and 7, after we have chosen our preferred parameter values in Section 4.

4 Parameter values

To solve for the general equilibrium of the model introduced in Section 3, we need to settle on plausible values for its parameters $\{\alpha^U, \beta, \omega^U, \theta^U, \tau^U, \bar{a}^U, \tilde{c}^U, \bar{S}^U, \bar{U}, r^a\}$. Because of the critical role the height elasticities $\{\omega^U, \theta^U\}$ play in shaping the equilibrium outcomes of the model and the related literature is in its infancy, we provide some estimates in Sections 4.1 and 4.2. Readers who are primarily interested in the application of the model may jump to Section 4.3 in which we discuss our preferred parameter values.

4.1 Costs of height

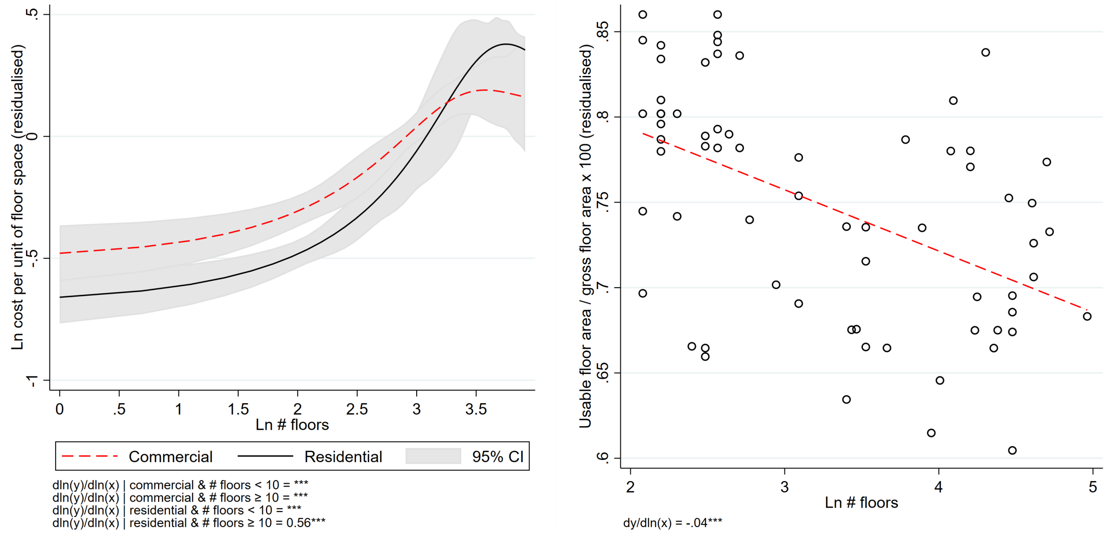
It is well established in the literature that the cost of supplying floor space increases in the density of development owing to diminishing returns to non-land inputs (Combes et al., 2021). In this section, we provide novel estimates of the use-specific elasticity of construction cost with respect to height θ^U and explore how innovations in construction technology may have affected the cost of height. For a review of the related literature, as well as a discussion of potential for future research, we refer to Appendix Section L.

[15] Regulation may result in a situation in which the profit-maximizing height, $S^*(x)$, is not realized and returns to height exceed the cost of height at the margin (Glaeser et al., 2005).

4.1.1 Estimating the cost of height

Using construction costs of a global cross-section of buildings provided by Emporis.com, we illustrate how the per-unit construction cost of commercial and residential buildings changes in height in Figure 6, panel (a). Evidently, taller buildings are more expensive to build in per-floor space terms as the height elasticity of per-unit construction cost is positive.[16] Moreover, the elasticity increases in height. For buildings up to a height of nine floors, we estimate a height elasticity of 0.1. Beyond that point, the cost of height increases significantly.

Figure 6: Cost of height



(a) Unit cost vs. height (b) Usable floor area vs. height

Note: In panel (a), we first regress the log of the ratio of building construction cost over building floor space against decade fixed effects, country fixed effects, and number-of-floors fixed effects. The displayed non-linear functions are the outcome of locally weighted regressions of the estimated number-of-floor fixed effects against the number of floors, using a Gaussian kernel and a bandwidth of 10. Confidence bands are at the 95% level. The vertical line marks the natural log of 10. Data are from from https://www.emporis.com/ (see Ahlfeldt and McMillen (2018) for details). In panel (b), each marker is an individual building. The dependent variable is residualized in regressions against country fixed effects and a time trend. The vertical line marks the natural log of 10. Observations were culled from Sev and Ozgen (2009), Watts et al. (2007), Kim (2004), and Berger (1967).

Panel (a) in Figure 6 not only suggests that the height elasticity of construction cost increases in height, but also suggests heterogeneity across uses. For taller buildings, we estimate a height elasticity of 0.26 for commercial buildings and about twice as large for residential buildings at 0.56. There are several reasons that may account for this difference. Residential units require more rooms with windows and, therefore, typically have smaller floor plates. Moreover, they need more sophisticated plumbing since residential units are

[16]We obtain the estimate of the height elasticity of construction cost from a regression of the log of construction cost per floor space unit (stripped of country and decade fixed effects) against the log of floor count.

equipped with bathrooms. Often, they have more complex facades with balconies (Smith et al., 2014).

There are few precedents in the literature to which we could compare our results. Most closely related to our estimates of the cost of height are Ahlfeldt and McMillen (2018) who also exploit the Emporis data set. Employing a more restrictive parametric estimation approach and extending the sample to very tall buildings, they estimate somewhat larger height elasticities of construction costs. Outside economics, there is a literature that provides engineering cost estimates. The rule of thumb is that construction costs tend to increase by 2% per floor (Department of the Environment, 1971), which is in line with more recent estimates (Tan, 1999; Lee et al., 2011).

4.1.2 Quantifying the effects of innovation on the cost of height

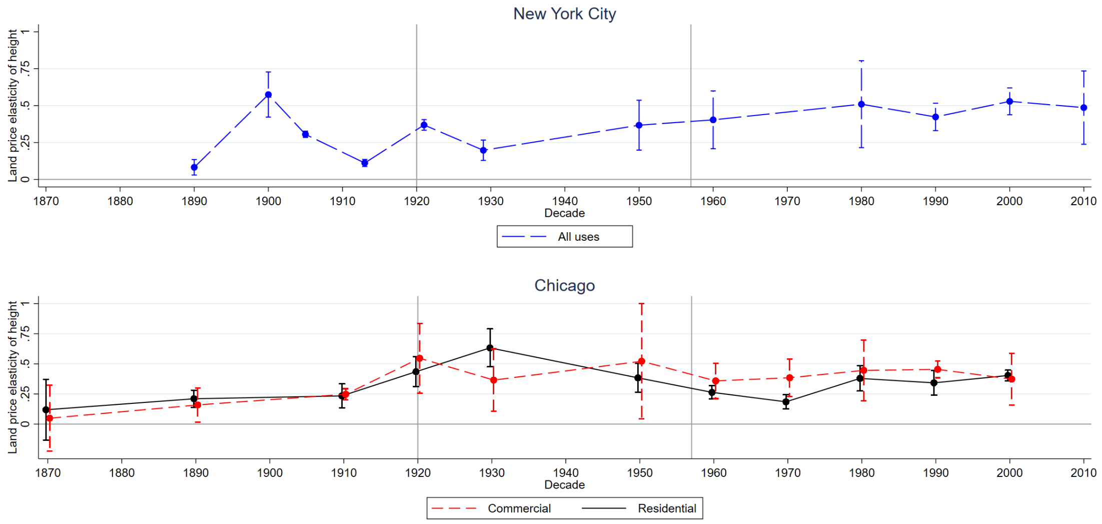
As discussed in some detail in Appendix Section J.1, the history of skyscrapers is marked by technological innovations. Quantifying the effect of technological innovations on the cost of height is empirically challenging since historical records of construction costs are not easily accessible. Within the structure of our model, the height elasticity of construction cost, θ^U, maps directly to the elasticity of height with respect to land rent $\kappa^U = \frac{\partial \ln S^U}{\partial \ln r^U} = \frac{1}{1+\theta^U}$ (see Appendix Section K.3 for a derivation). In a more general framework, Ahlfeldt and McMillen (2018) obtain $\kappa^U = \frac{\partial \ln S^U}{\partial \ln r^U} = \frac{\sigma^U}{1+\theta^U-\lambda^U}$. Our model features a special case in which the elasticity of substitution between land and capital is $\sigma^U = 1$ and the fraction of the parcel that is developable is independent of height (the height elasticity of extra space is $\lambda^U = 0$).

In Figure 7, we provide estimates of κ^U for New York City and Chicago at different points in time from 1870 to 2010. Following Ahlfeldt and McMillen (2018), each estimate is derived from a building-level regression of the log of building height against the log of land value, using distance from important points of interest as instruments.[17] In line with our theoretical expectation, we find that the elasticity estimates increase over time. It is noteworthy that there were major reforms in the zoning regimes in Chicago in 1920, 1923, and 1957, which likely affected the relationship between building heights and land rents. Unlike for Chicago, the trend for New York is quite smooth across those dates.

We use our estimates of κ^U to recover decade-specific estimates of $\theta^U = \frac{\sigma^U-(1-\lambda^U)\kappa^U}{\kappa^U}$ under the more restrictive parametrization from our model ($\sigma^U = 1, \lambda^U = 0$), as well as borrowing parameter values $\sigma^C = 0.66, \sigma^R = 0.61, \lambda^C = 0.15, \lambda^R = 0.1$ from Ahlfeldt and McMillen (2018). We then regress the log of θ^U against a trend variable, weighting observations by the inverse of the standard errors in Figure 7. Our tentative interpretation of the results is that the cost of height decreased by about 2% per year over the course of the 20th century, although we stress that this interpretation hinges on assuming constant values for the elasticity of substitution between land and capital and the height elasticity of extra space. We refer to Appendix Section L.1 for further details.

[17] We refer to Figure 7 notes for additional detail on the methodology and underlying data.

Figure 7: Elasticity of height with respect to land price in New York and Chicago



Note: Each point estimate is from a separate regression of the log of height of buildings constructed over a decade against the log of land value at the beginning of the decade, using the following instrumental variables. Log distance from Empire State Building and log distance from Wall Street for New York; Log distance from CBD and log distance from Lake Michigan for Chicago. 1940 missing in both data sets due to lack of completions. 1880 and 1900 missing for Chicago due to missing land value data. 1970 effect for New York is an imprecisely estimated outlier that was dropped to improve readability. Chicago data are from Ahlfeldt and McMillen (2018). New York city land values are from Barr (2016) and Spengler (1930). New York building height data are from the 2018 NYC PLUTO file from the NYC Dept. of City Planning.

4.2 Returns to height

Traditional urban economics models focus on how accessibility leads to variation in firm profits and household utility that capitalizes in horizontal rent gradients. However, productivity and amenities may also vary within buildings, leading to vertical rent and density gradients. In the model introduced in Section 3, developers face returns to height that are monitored by ω^U, the height elasticity of floor space rent. In this section, we provide estimates of ω^U and explore whether there is a time trend. For a review of the related literature, as well as a discussion of potential for future research, we refer to Appendix Section M.

4.2.1 Estimating returns to height

The commercial and residential bid rent functions derived in Section 3 establish a positive relationship between rent and height. It is straightforward to estimate ω^U by regressing the log of rent against the log of the floor at which a unit is located within a building, controlling for arbitrary building (and location) characteristics via building fixed effects.[18] Figure 8 illustrates how the per-unit rent changes across floors within tall buildings in New York City and Chicago.[19] Evidently, there is a positive vertical height gradient. For

[18]See Appendix Section M.1.1 for a derivation of an estimation equation.

[19]We refer to Figure 8 for a description of the underlying data.

commercial units, the positive height gradient can originate from increased revenues due to a signaling of quality to customers or a lower wage bill if height is a workplace amenity for which workers are willing to accept a compensating wage differential (Liu et al., 2018). For residential units, the rent increase likely reflects an amenity effect, e.g., due to better views. Note that we use purchase prices to mitigate effects of rent regulation, implicitly assuming that prices map to rents via a constant capitalization rate.

Reassuringly, we estimate an almost identical residential height elasticity of rent of about 0.07 for both cities.[20] For New York City, where we also have access to commercial rent data, we estimate a slightly lower commercial height elasticity of rent of 0.03. The somewhat lower height elasticity of rent for commercial than for residential buildings is consistent with luxurious condominiums occupying the upper floors in mixed-used skyscrapers, such as the Woolworth Building in New York or the Shard in London.[21]

Our estimates connect to a small literature that has estimated vertical height gradients within buildings, i.e., conditional on building fixed effects. For two samples of commercial buildings that cover various U.S. cities, Liu et al. (2018) estimate significantly larger values of the height elasticity of unit rent of 0.086 within the CompStat data set and even 0.189 within their Offering Memos data set. As for residential height premiums, Danton and Himbert (2018) estimate that within buildings in Switzerland, residential rents increase by 1.5% per floor. Their sample mostly consists of smaller structures. For the average floor of two within their sample, the implied height elasticity of unit rent is 0.03, somewhat less than what we find for taller residential structures in New York City and Chicago. One conclusion from relatively few existing estimates exploiting within-building variation is that the height elasticity of rent varies significantly across samples, land use, and, as we discuss next, time.

There are more estimates of the effect of the floor at which a unit is located on prices and rents from specifications that do not control for building fixed effects. While there is the concern that unobserved location attributes and within-building spillovers may confound the height effect, the results in this literature substantiate that the height elasticity of rent is positive (see Appendix Section M.2).[22]

It is worth recalling that in the context of our model, the height elasticity of rent ω^U is a reduced-form parameter that depends on the height elasticity of amenity $\tilde{\omega}^U = (1-\alpha^U)\omega^U$. Assuming the canonical values $1 - \alpha^C = 0.15$ (Lucas and Rossi-Hansberg, 2002) and $1 - \alpha^R = 0.33$ (Ahlfeldt and Pietrostefani, 2019), our estimates of ω^U imply that the

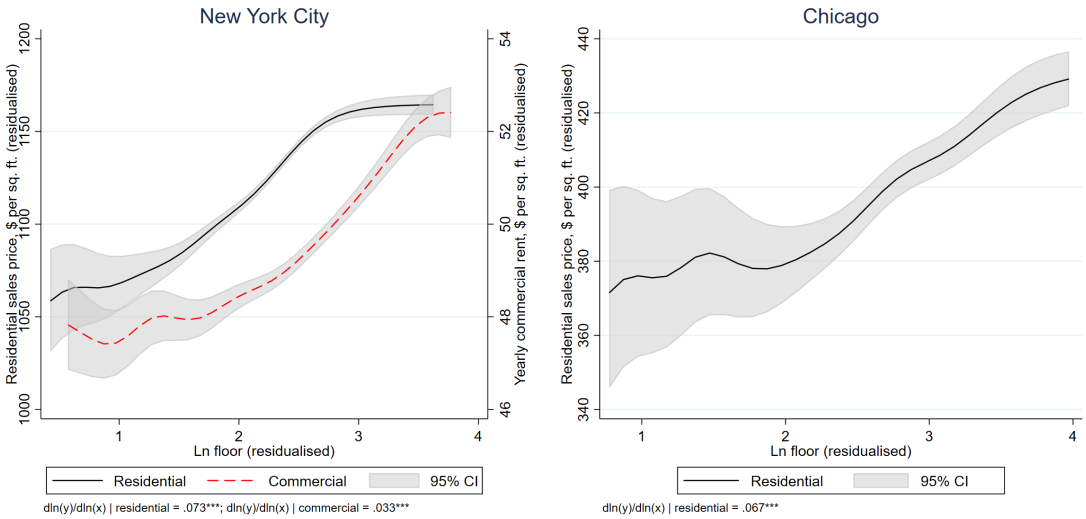
[20]We obtain all elasticity estimates from regressions of log of rent or price against the log of the floor, controlling for unit characteristics, building fixed effects and time effects.

[21]The conclusion that returns to height are greater within residential than commercial buildings rests on the assumption that the capitalization rate of rents is independent of the floor.

[22]It is worth pointing out that, for marginal changes, the height elasticity of unit rent discussed here and the height elasticity of average building rent derived in Eqs. (2) and (4) are the same. In reality, changes in height are typically non-marginal due to the integer floor constraint. In Appendix Section M.1.2, we show that for smaller buildings, suitable values of the height elasticity of average building rent that feed into the developer problem in Eq. (5) will typically be smaller than the values of the height elasticity of unit rent reported here.

productivity of firms, within a given building, increases in height at an elasticity of 0.005. Likewise, residential utility increases in height at an elasticity of 0.025.

Figure 8: Returns to height

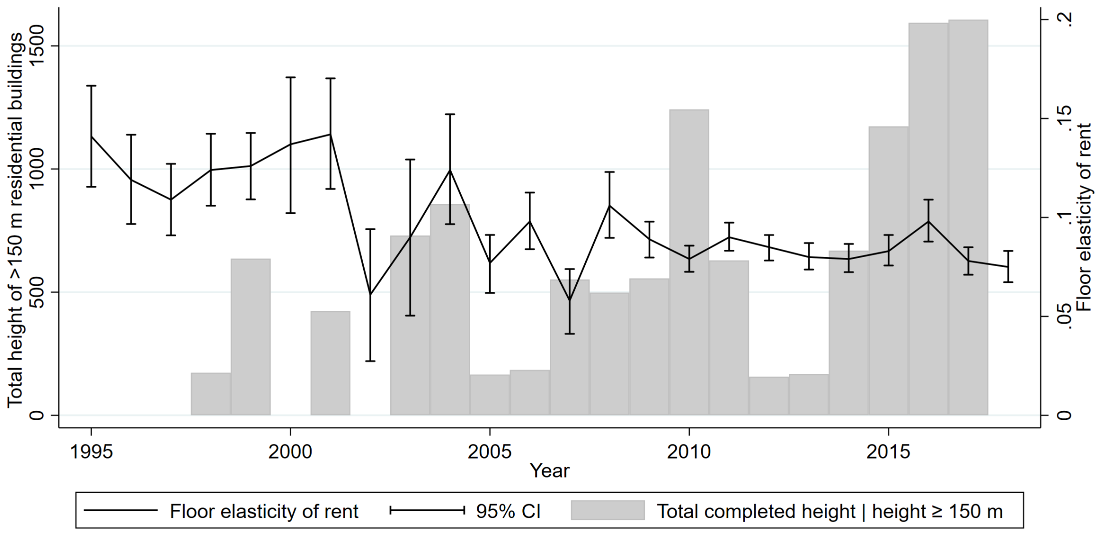


Note: Price, rent and floor are residualized in regressions (in logs) against unit characteristics and building fixed effects and time effects. The displayed non-linear functions are the outcome of locally weighted regressions using a Gaussian kernel and a bandwidth of 0.2. Confidence bands are at the 95% level. We trim the data set to exclude outliers that fall into the bottom or top percentile in terms of residualized ln rent/price or residualized ln floor before estimating the non-parametric gradient to improve the presentation. The parametric elasticity estimates are from the full sample. Residential prices are from StreetEasy. Commercial asking rents are from Cushman & Wakefield.

4.2.2 Returns to height over time

Little is known about how the height elasticity of rent has changed over time. Figure 9 summarizes year-specific estimates of the residential height elasticity of rent over 22 years in New York City. Since the early 2000s, there has been a remarkable decline in the returns to height that is seemingly at odds with the notion that skilled workers increasingly demand amenities (Glaeser et al., 2001). However, it is quite notable that the decrease in the height premium coincides with an expansion of the supply of tall residential buildings. The total height of completed residential buildings that are 150 meters or taller sharply increased around the turn of the century. If there is heterogeneity in preferences, (implicit) prices are generally not independent of supply. One interpretation of the patterns observed in Figure 9 is that there is significant dispersion in tastes for height. As the availability of the height amenity has increased, the valuation of the marginal buyer may have decreased. Alternatively, the amenity value of living on a high floor could have diminished since views are being obscured by additional tall buildings.

Figure 9: Returns to height over time in the New York City residential market



Note: The floor elasticity of rent is estimated in regressions of log rent against log floor by year, controlling for unit characteristics and building fixed effects. Confidence bands are at the 95% level. Residential rents are implied rents based on apartment prices data from StreetEasy, assuming a constant discount rate. Total completed height of residential buildings exceed 150 meters are from the CTBUH Skyscraper Center.

4.3 Baseline parameter values

Having reviewed the evidence on costs and returns to height in Sections 4.1 and 4.2, we are now ready to set the parameter values of the model developed in Section 3. We summarize our parameter choices in Table 1, where we also provide references to the related literature. These parameter values are not taken from individual papers and do not necessarily correspond to our own estimates. Instead, they represent what we view as canonical values that are are suitable for stylized presentations and simple counterfactual analysis. We wish to highlight that the evidence base on the costs of and returns to height is small and parameter estimates of $\{\omega^U, \theta^U\}$ vary across samples and periods. Hence, some uncertainty surrounds our choices. That said, we provide various over-identification tests throughout the following sections that reinforce our belief that the chosen parametrization is sensible.

We use canonical values for the share of floor space as inputs $(1-\alpha^C)$, the share of floor space at consumption $(1-\alpha^R)$, and the agglomeration elasticity, β. We choose values of the height elasticity of construction cost that are in the ballpark of the estimates reported by Ahlfeldt and McMillen (2018), taking into account that the residential elasticity, θ^C, tends to be larger than the commercial elasticity, θ^R, (see Section 4.1). We acknowledge that the height elasticity of construction costs is likely increasing in height. Since we use the model for stylized predictions and illustrative counterfactual analyses, we stick to the parameterization with a constant elasticity for tractability. For the height elasticity of rent, we use our estimates reported in Section 4.2, taking into account that the residential elasticity, ω^R, appears to be larger than the commercial elasticity, ω^C. For the spatial decay in production amenity, τ^C, we choose a value based on novel estimates of the commercial rent gradient across a set of 55 global cities that complement recent evidence from the U.S. (Rosenthal et al., 2021) and are reported in Appendix Section N.1.1. This decay in production spillovers is conceptually different from the one estimated by Ahlfeldt et al. (2015), who assume that each location emanates spillovers to nearby areas. Guided by Ahlfeldt et al. (2015), we choose a smaller value for the residential amenity decay, τ^R.

Table 1: Parameter values

	Parameter	Value	Further reading
$1-\alpha^C$	Share of floor space at inputs	0.15	Lucas and Rossi-Hansberg (2002)
$1-\alpha^R$	Share of floor space at consumption	0.33	Combes et al. (2019)
β	Agglomeration elasticity of production amenity	0.03	Combes and Gobillon (2015)
θ^R	Commercial height elasticity of construction cost	0.5	Ahlfeldt and McMillen (2018)
θ^R	Residential height elasticity of construction cost	0.55	Ahlfeldt and McMillen (2018)
ω^C	Commercial height elasticity of rent	0.03	Liu et al. (2018)
ω^R	Residential height elasticity of rent	0.07	Danton and Himbert (2018)
τ^C	Production amenity decay	0.01	Ahlfeldt et al. (2015)[a]
τ^R	Residential amenity decay	0.005	Ahlfeldt et al. (2015)

Notes: These parameter values are not taken from individual papers and do not necessarily correspond to our own estimates. Instead, they represent what we view as canonical values that are suitable for stylized presentations and simple counterfactual analysis. The last column provides a references for the interested reader for further reading, but not necessarily the source of a point estimate. [a]The parameter value is consistent with the commercial rent gradient estimated for a large set of global cities, assuming $\alpha^C = 0.15$ (see Appendix Section N.1.1). We set the following scale parameters arbitrarily to to generate a plausible land use pattern: $\bar{a}^C = 2, \bar{a}^R = 1, \tilde{c}^C = 1.4, \tilde{c}^R = 1.4, r^a = 30, \bar{U} = 1$. There are no binding height limits in the baseline parametrization ($\bar{S}^C = \bar{S}^R = \infty$).

5 Vertical and horizontal city structure

In this section, we solve our model to derive the vertical and horizontal structure of a stylized city. We contrast the predictions for the height gradient with novel evidence from a large set of cities around the world. For a review of the related literature, as well as a discussion of potential for future research, we refer to Appendix Section N.

5.1 Stylized gradients

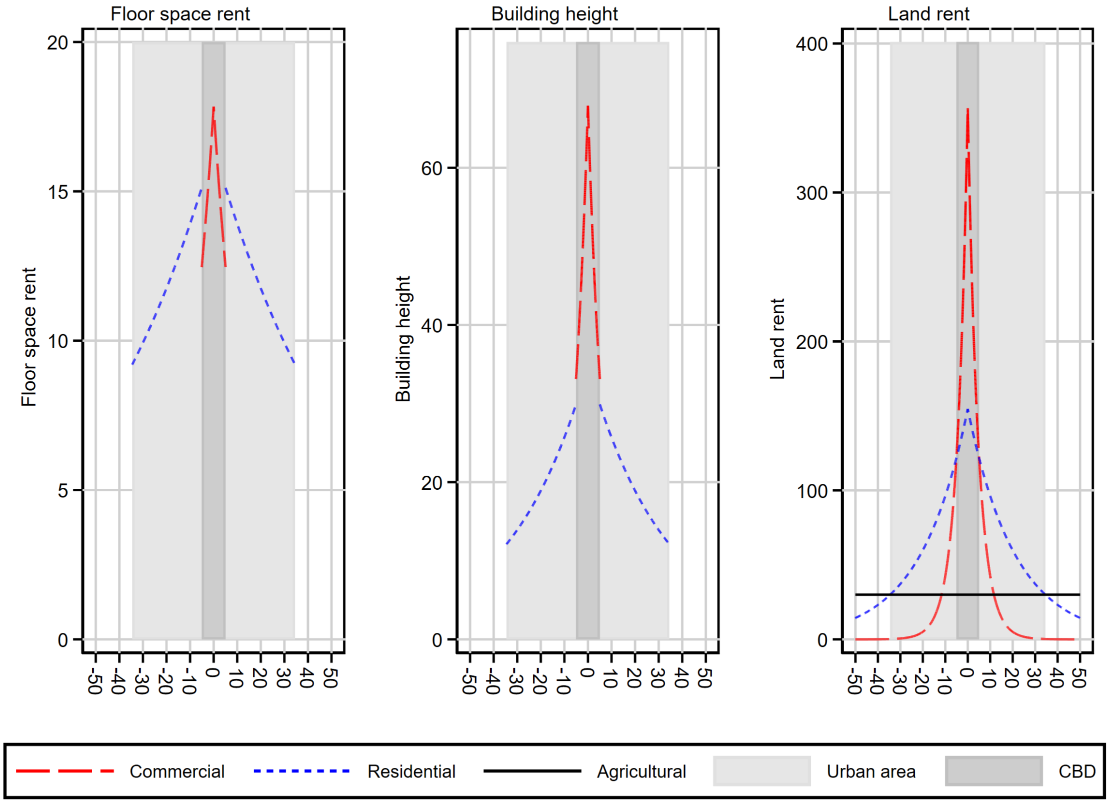
Solving the model under the parameter values defined in Table 1 delivers the gradients illustrated in Figure 10. The floor space rent for both uses naturally decrease in distance from the historic city center to compensate users for the associated loss of amenity. This is a standard prediction of spatial equilibrium models of internal city structure ever since Alonso (1964), if not von Thuenen (1826). We find that the commercial rent gradient is steeper than the residential rent gradient. An inspection of the first derivative of the log of floor space rent with respect to distance from the core $\frac{\partial \ln \bar{p}^U(x)}{\partial D(x)} = -\frac{\tau^U \theta^U}{(1-\alpha^U)(\theta^U - \omega^U)}$ reveals that the difference in the slope of the gradients is driven as much by the difference in amenity decay ($\tau^C > \tau^R$) as by the greater ease at which firms substitute away from floor space ($1-\alpha^C < 1-\alpha^R$). Due to the effect of the height amenity, the rent gradient also depends on the realized building height and, hence, the height elasticities $\{\theta^U, \omega^U\}$, which is a novel theoretical insight. The chosen parameter values imply a semi-elasticity of building rent with respect to distance from the historic center of -0.071 for commercial use and -0.017 for residential use.

Higher floor space rents imply greater marginal returns to height and greater profit-maximizing building heights. Therefore, the height gradient follows the floor space gradient qualitatively. This is the standard prediction of neoclassical urban models incorporating a housing supply side (Mills, 1967; Muth, 1969), which has motivated Ahlfeldt and Barr (2020) to argue that tall historic buildings serve as proxies for historic centers that are often difficult to detect. The first derivative $\frac{\partial \ln S^{*U}}{\partial D} = -\frac{\tau^U}{(1-\alpha^U)(\theta^U-\omega^U)}$ reveals that the height gradient will be steeper than the floor space gradient unless the height elasticity of construction cost is set to implausibly large values ($\theta^U \geq 1$). It is intuitive that the slope of the height gradient depends on the use-specific height elasticities $\{\theta^U, \omega^U\}$. The chosen parameter values imply a semi-elasticity of height with respect to distance from the historic center of -0.142 for commercial use and -0.032 for residential use.

The developer's ability to multiply land to floor space via height results in land bid rents that increase more than proportionately in floor space rents. Hence, Figure 10 illustrates how tall buildings rationalize extreme differentials in land prices across small areas, such as documented by Ahlfeldt and McMillen (2014) and Barr (2016). In fact, we generate an elasticity of land rent with respect to distance from the historic center of -0.53, which is within the range of estimates that Ahlfeldt and Wendland (2011) and Ahlfeldt and McMillen (2018) find for Berlin and Chicago (see Appendix Section N.1.2 for details).

Because land is allocated to the highest bidder, we can derive the horizontal land use pattern from the right panel of Figure 10. The dark-shaded area where the commercial bid-rent for land exceeds the residential bid-rent marks the central business district (CBD). Similarly, the light-shaded area marks the residential zone where returns to residential development exceed the opportunity return in commercial and agricultural use. A novel prediction of our model is that, via their effect on floor space bid rents and profit-maximizing building heights, use-specific costs and returns to height affect the commercial and residential land rents and, hence, the horizontal land use pattern. This way, differences in the height elasticities $\{\theta^U, \omega^U\}$ across uses have interesting feedback effects on the use-specific levels of floor space rents and building heights. An interesting feature of our parameterization is that the height and floor space gradients are discontinuous at the land use border. Since $\theta^R - \omega^R > \theta^C - \omega^R C$, developers face greater net costs of height when developing residential rather than commercial buildings. Therefore, developers of commercial buildings just inside the CBD can afford to build taller at lower floor space rents than developers of residential buildings just outside the CBD. Indeed the discontinuities disappear if we set $\theta^R = \theta^C$ and $\omega^R = \omega^C$ (see Appendix Section N.1.3).

Figure 10: Urban gradients

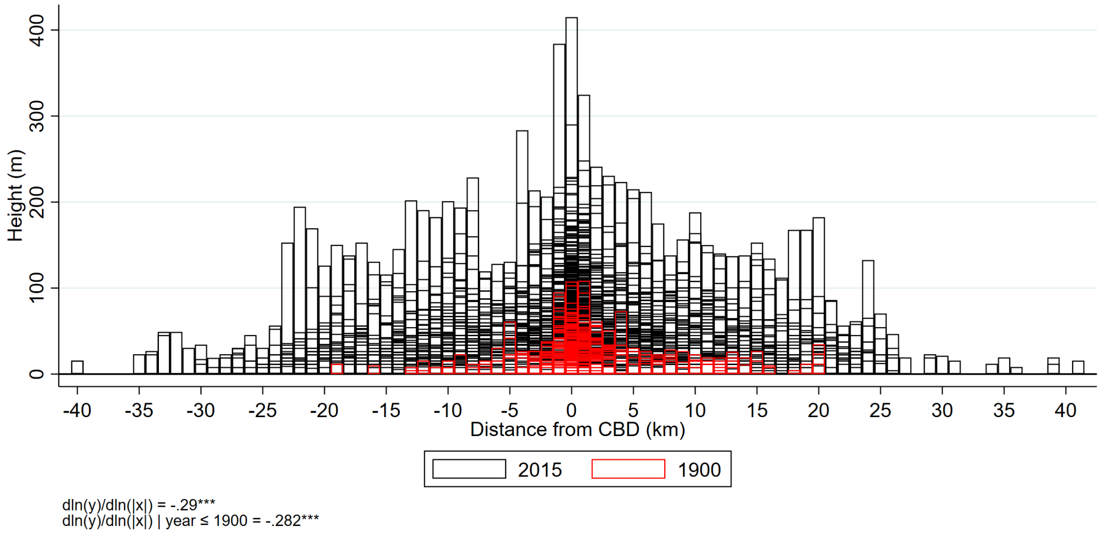


Note: The figure illustrates the solution to the model laid out in Section 3 using the parameter values reported in Table 1. Floor space rent is the average per-unit rent within a building $\bar{p}^U(x)$. X-axis gives the distance from the historic center in units that roughly correspond to kilometers in real-world cities.

5.2 Evidence on height gradients

In Figure 11, we pool the urban height profile of 55 North American cities sampled by Ahlfeldt et al. (2020). Consistent with the predictions of our model, building heights decrease relatively steeply in distance from the centre of the CBD. While, of course, absolute heights increased significantly, our estimates of the elasticity of height with respect to distance from the CBD, at -0.29, are almost identical for buildings that existed in 2015 and in 1900. In Table 2, we further zoom out to cover all 125 global cities for which Ahlfeldt et al. (2020) identify "global" prime locations (our CBD proxy). We focus on the tallest building within a one-kilometer distance ring from the CBD since we presumably measure the height of the tallest building with less error than the height of the average building. A comparison of columns 1 and 2, again, reveals a stability of the slope of the height gradient that is remarkable given the significant reductions in the cost of height suggested by the results in Section 4.1. One interpretation of the evidence is that, over the 20$^{\text{th}}$ century, reductions in the cost of height (monitored by θ^U) and the cost of CBD access (monitored by τ^U) have just about offset each other in their impact on the (relative) height gradient.

Figure 11: Tallest buildings by North American city, and distance from the CBD



Note: Each bar illustrates the height of the tallest building within a one-km bin to the west or the east of the CBD in one of 55 North American cities. Height data from Emporis. CBDs are the "global prime locations" identified by Ahlfeldt et al. (2020). Negative (positive) distance values indicate a location in the west (east) where the x-coordinate in the World Mercator projection is smaller (larger) than the x-coordinate of the CBD. Average height elasticities estimated conditional on city fixed effects. Data from https://www.emporis.com/ (see Ahlfeldt and McMillen (2018) for details).

In columns (3) and (4) of Table 2, we find that the commercial height gradient is steeper than the residential height gradient. This finding is consistent with the model predictions under the chosen parametrization and can be rationalized by a relatively steep decay in the production amenity, a low input share of floor space in production, or a relatively low net cost of height in commercial development. A more specific prediction of the model is that the height gradient will be discontinuous if (and only if) costs and returns to height differ by land use. To test this prediction, we identify the edge of the CBD as the distance at which the residential building density starts exceeding the commercial building density in 15 large U.S. cities. Using a boundary discontinuity design, we estimate that building heights, on average, increase by 0.2 log points as one enters the CBD (see Appendix Section N.2 for details). This result provides indirect evidence for a relatively lower commercial height elasticity of construction cost, adding to the direct evidence in Section 4.1. The remaining columns of Table 2 illustrate how the height gradient is steeper in North American cities than in European or Asian cities. This suggests a role for height regulation, which we discuss in Section 7.

5.3 Fuzzy height gradient

Except for the height discontinuity at the land use boundary, all gradients in Figure 10 are smooth. This is because we have summarized the amenity value of location by a smooth function of distance from the city center. This is a stark abstraction of vertical global cities

Table 2: Height gradient estimates

	(1) Ln(height)	(2) Ln(height)	(3) Ln(height)	(4) Ln(height)	(5) Ln(height)	(6) Ln(height)	(7) Ln(height)
Distance from	-0.069***	-0.042***	-0.052***	-0.036***	-0.024***	-0.023***	-0.048***
CBD center (km)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
City fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$d\ln(y)/d\ln(x)$	-.23	-.27	-.33	-.26	-.12	-.12	-.35
Building Sample	$t^a \leq 1900$	All	Comm.[b]	Residential	Asia	Europe	North A.[b]
Observations	344	3469	1294	1185	2081	662	419
R^2	.494	.559	.679	.647	.427	.331	.376

Notes: [a]: Completion year. [b] Commercial. [c]: North America. Unit of observation is city-distance bin (1 km). Height is the height of the tallest building within a one-km distance bin. Building data from Emporis. CBD definitions (global prime locations) for 125 global cities from Ahlfeldt et al. (2020). Elasticities computed at the sample means. Data from Empiris (see Ahlfeldt et al. (2020) for details). * p < 0.1, ** p < 0.05, *** p < 0.01.

where amenity values can vary remarkably within short distances due to scenic views, noise pollution, or access to subway stations. Since all (dis)amenities eventually capitalize into floor space rents, the profit-maximizing height can vary significantly over short distances, consistent with tall buildings standing side by side with smaller structures. An interesting question is whether our model, under the parameter values that we deem canonical, can rationalize real-world skylines with plausible variation in amenity values.

To answer this question, we follow the conventions in the literature on quantitative spatial models of internal city structure (Ahlfeldt et al., 2015) and i) discretize space, ii) introduce a location-use-specific structural fundamental that scales the amenity parameters $\{\bar{a}^C, \bar{a}^C\}$, and iii) exploit the direct mapping from amenity to height established in Eq. (6) to invert the structural fundamental from building heights observed in data. Intuitively, we solve for the unobserved values of the fundamental amenity that make the observed values of building heights consistent with an equilibrium in our model.

We take Chicago as a case in point; most tall buildings are approximately linearly aligned along the shore of Lake Michigan. To create a stylized version of the Chicago skyline, we first generate a linear grid of 100-meter cells defined in terms of the northings (the y-coordinate in a projected grid reference system). The equivalent of x in our model is then simply the difference of the northing of a grid cell to the northing of the global prime location in Chicago, as identified by Ahlfeldt et al. (2020). We assign the height of the tallest building and its use observed in the Emporis data to the entire grid cell. The resulting stylized skyline, which we exactly match in our model, is in the upper panel of Figure 12. Evidently, the model now accommodates a *fuzzy height gradient* with substantial micro-geographic variation in building heights. We also observe mixing of uses because of tall residential buildings located within the CBD where there are direct views on amenities, such as Lake Michigan or the Chicago River. Towards the edges of the simulated city, there are empty parcels, which we rationalize with a zero fundamental amenity in a theory-consistent way.

In the bottom-left panel, we illustrate the distribution of the fundamental amenity values that convert the smooth height gradients in Figure 10 into the real-world fuzzy height gradient in the top panel of Figure 12. The residential fundamental amenity scales utility by factors that range between about 1 and 1.3, with the great majority of locations falling into a ± 0.1 window centered on 1.1. The commercial fundamental amenity scales productivity by factors that fall into an even narrower range of about 0.85 to 1. Hence, relatively little variation in fundamental amenity is required to introduce sizable fuzziness into the height gradient. Intuitively, users substitute away from floor space as its price increases, leading to a more-than-proportionate increase in floor space rents as the amenity value increases. As the floor space rent increases, developers use relatively more capital to construct buildings whose height increases more than proportionately in floor space as long as $\{\theta^U - \omega^U\} < 1$. Therefore, substitution on the consumption side (by users) and production side (by developers) amplify small differences in amenity into large differences in height and land rent.

The bottom-right panel suggests that our model does a good job in describing these substitution patterns. Given the fundamental amenity values inverted from building heights, the model generates land rents that scale proportionately to land values observed in data (from Ahlfeldt and McMillen, 2018). The fit along the 45-degree line suggests that the model not only correctly predicts land rents to be higher where they tend to be higher in the real world, but also correctly predicts the relative magnitudes. This lends support to the chosen parametrization and substantiates the point made by Ahlfeldt and Barr (2020) who argue that tall historic buildings can be used to predict centres of economic activity in history.

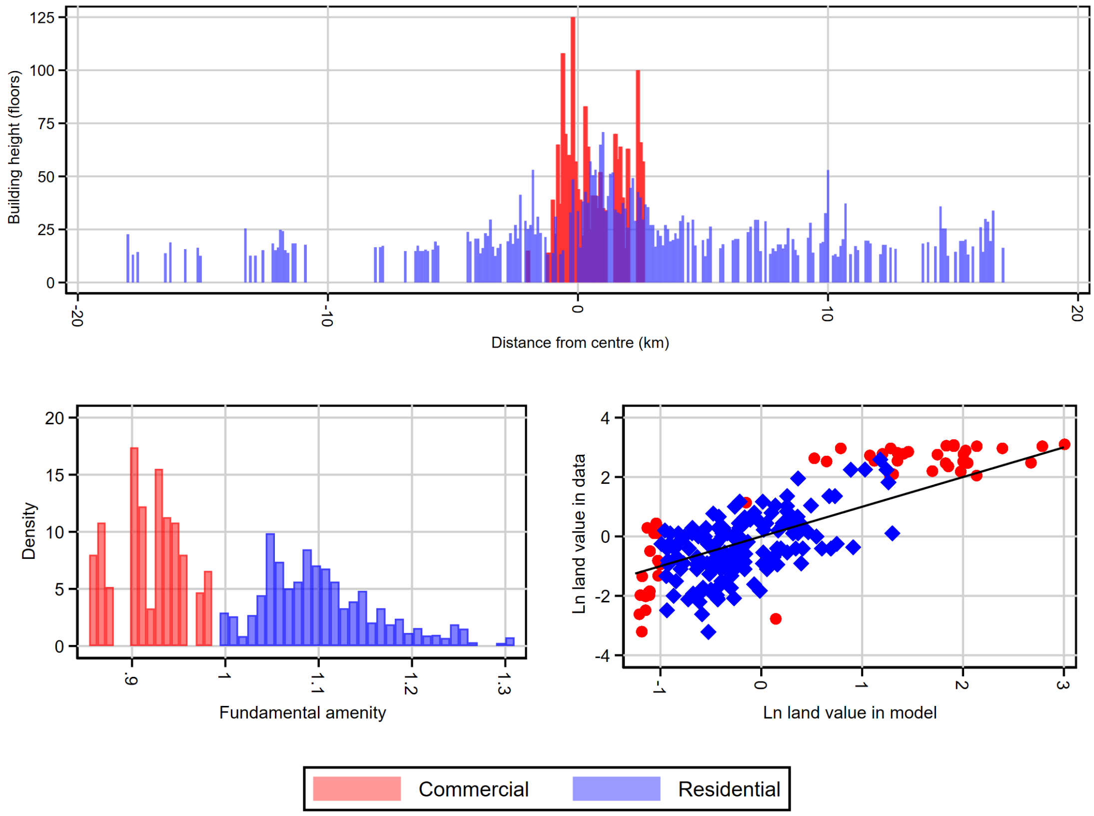
The main conclusion from Figure 12 is that relatively small differences, with respect to the micro-geographic amenity, rationalize large differences in heights of adjacent buildings, especially if there is mixed land use. Of course, there are additional explanations which we discuss in more detail in Appendix Section N.4.[23] Yet, a simple model can account for the shape of real-world skylines largely through differences in commercial and residential amenities, an important lesson for future quantitative spatial models that tackle the vertical dimension of cities.

6 Skyscrapers as causes and effects of agglomeration

Empirically, skyscrapers appear to be a distinctively urban phenomenon (see Section 2). Therefore, it is surprising that little research has been done to establish the relationship between urbanization and vertical growth in a systematic and robust manner. In this section, we explore theoretically and empirically how cities expand into the vertical di-

[23] Fragmentation of land ownership can prevent the assembly of sufficiently large lots to develop a skyscraper (Strange, 1995; Brooks and Lutz, 2016). The durability of tall structure can add to the fuzziness of skylines that consist of skyscrapers developed at different points in time (Brueckner, 2000), although we do not find the fuzziness of the height gradient is much reduced within construction date cohorts (see Appendix N.3). Similarly, delayed skyscraper development generated by option values (Titman, 1985; Williams, 1991) can lead to fuzziness. Finally, there is a battery of institutional factors, such as land use regulation, historic preservation, and public ownership, that can add to micro-geographic variation in building heights.

Figure 12: Fuzzy height gradient



Note: Figure illustrates the solution to the model laid out in Section 3 using the parameter values reported in Table 1), discretized space (to 100-meter cells) and a multiplicative structural amenity components that we invert such that the model matches the stylized skyline of Chicago in the top panel. Distance from the center is the difference between the northings of a grid cell and the northing of the global prime location identified by Ahlfeldt and McMillen (2018). The bottom-left panel shows the distribution of the inverted fundamental amenity. The bottom-right panel compares the model's predictions of land rent to 2000 land values from Ahlfeldt and McMillen (2018).

mension as the level of agglomeration increases and reductions in the cost of height foster agglomeration. For a review of the related literature, as well as a discussion of potential for future research, we refer to Appendix Section O.

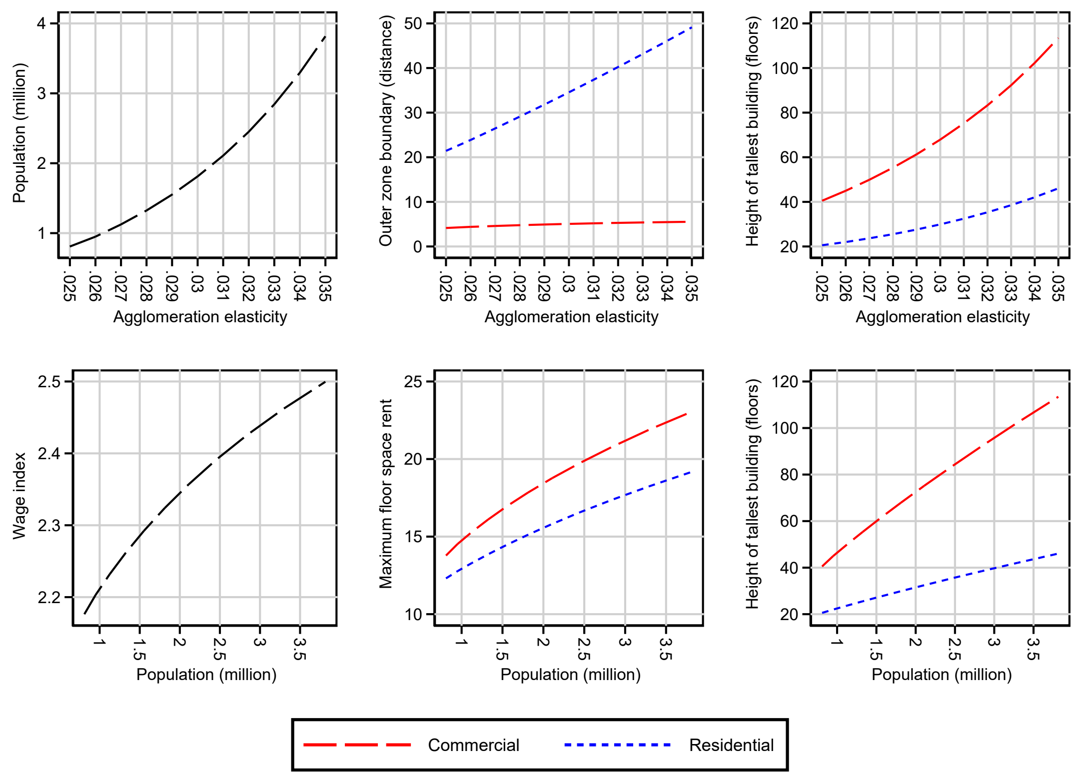
6.1 Returns to agglomeration

One reason why cities exist is that agglomeration leads to greater productivity (Combes and Gobillon, 2015). Returns to agglomeration have likely risen over the 20th century (Moretti, 2012); this should have led to increasing demand for central city locations and the development of skyscrapers.[24] We subject this intuition to a quantitative evaluation by solving the model developed in Section 3 for varying values of the agglomeration elasticity β.

[24] Autor (2019) documents that over the recent decades this trend applies to high-skilled workers, exclusively. In contrast, the urban wage premium in the U.S. has sharply declined for low-skilled workers since the 1970s.

We correlate some of the endogenous model outcomes with the set values of β in Figure 13. The left panels show how greater returns to agglomeration translate into higher wages, which leads to the attraction of a greater workforce. Because we follow the canonical spatial equilibrium framework and allow for free migration of homogeneous workers, the equilibrium population is highly sensitive to the choice of β. In response to increasing demand, the city expands horizontally (upper-middle panel) and vertically (right panels). Whereas the residential zone expands significantly into the former agricultural hinterland, the CBD grows primarily into the vertical dimension. Greater competition for space leads to higher equilibrium floor space rents since taller buildings are more expensive to build (bottom-middle panel). The city-size elasticity of floor space rent inferred from a comparison of equilibrium outcomes across simulation runs is about 0.3. This is what would be expected for a large and dense city according to the empirically derived rule-of-thumb by Ahlfeldt and Pietrostefani (2019), lending further support to our parametrization of the model.[25] This counterfactual exercise also establishes an elasticity to which we return with empirical estimates in Section 6.4: The city size elasticity of the tallest building height ranges from 0.52 (residential) to 0.66 (commercial). We refer to Appendix Section O.1 for further detail on those estimates.

Figure 13: Variation in external returns to agglomeration



Note: We report solutions to the model developed in Section 3 under varying values of the agglomeration elasticity β. All other parameter values are kept constant at the levels reported in Table 1.

[25] This value is also in line with estimates for large French cities (Combes et al., 2019).

6.2 Transport innovations

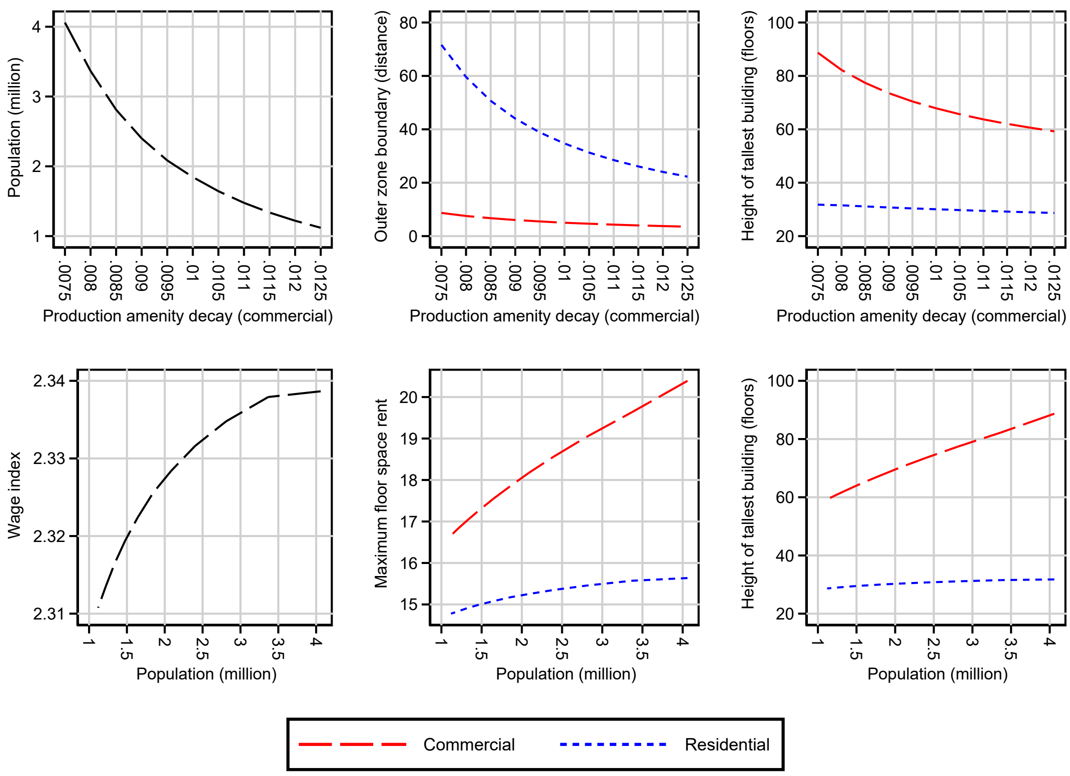
New transport technologies that started with horse-driven street cars, then improved to subways, and then automobiles and highways have significantly reduced within-city transport costs. It is well established in the literature that improvements in transport technology cause cities to grow and economic activity within cities to decentralize (Baum-Snow, 2007; Redding and Turner, 2015). Less is known about the implications on building heights. In our model, within-city transport cost is moderated by the amenity decay parameters $\{\tau^C, \tau^R\}$. As already discussed in Sections 3 and 5, smaller values of $\{\tau^C, \tau^R\}$ lead to a flatter height gradient. The implications for absolute building heights are less straightforward since the relative increase in attractiveness of peripheral locations and the increase of the attractiveness of the city as a whole have countervailing effects on building heights in central locations. To illustrate how these conflicting forces play out in our model, we solve the model for varying values of the amenity decays $\{\tau^C, \tau^R\}$. In each run, we set a different value for the commercial amenity decay τ^C and adjust the residential amenity decay such that the difference between the commercial and residential decay parameters conforms to the baseline, i.e. $\tau^R = \tau^C - 0.005$. This way, we only change one parameter value at a time, which allows us to present the results in Figure 14 using the same structure as in Figure 13.

As we increase transport costs, the city becomes less attractive. Hence, the population shrinks and wages fall, owing to lower agglomeration economies (left panels). In line with intuition, the city contracts in horizontal space (upper-middle panel) as transport costs increase. While the partial equilibrium effect would be a shift in demand towards the city center, resulting in an increase in floor space rents and building heights in central locations, the general equilibrium effect is the opposite. The city-wide decrease in population and wage more than offsets for the relative increase in demand for central locations, so that commercial building heights decrease (upper-right panel). The effect on residential heights is qualitatively similar, but quantitatively smaller since the adjustment in the horizontal size of the zone is much greater. Because the increase in population as we reduce transport cost is driven by a horizontal expansion, we find relatively low city size elasticities of rent and building height. The city size elasticity of rent ranges from 0.05 (residential) to 0.15 (commercial). The city size elasticity of tallest building height ranges from 0.08 (residential) to 0.3 (commercial). We refer to Appendix Section O.1 for further detail on those estimates.

6.3 Construction technology

Various technological innovations discussed in Section 2 and Appendix Section J.1 have led to a reduction in the cost of constructing tall buildings since the end of the 19$^{\text{th}}$ century. The results in Section 4.1 point to a significant reduction in the height elasticity of construction cost. To evaluate the effects of this important, yet understudied, technological trend on urban structure, we solve our model for varying values of the height elasticity of

Figure 14: Variation in transport technology

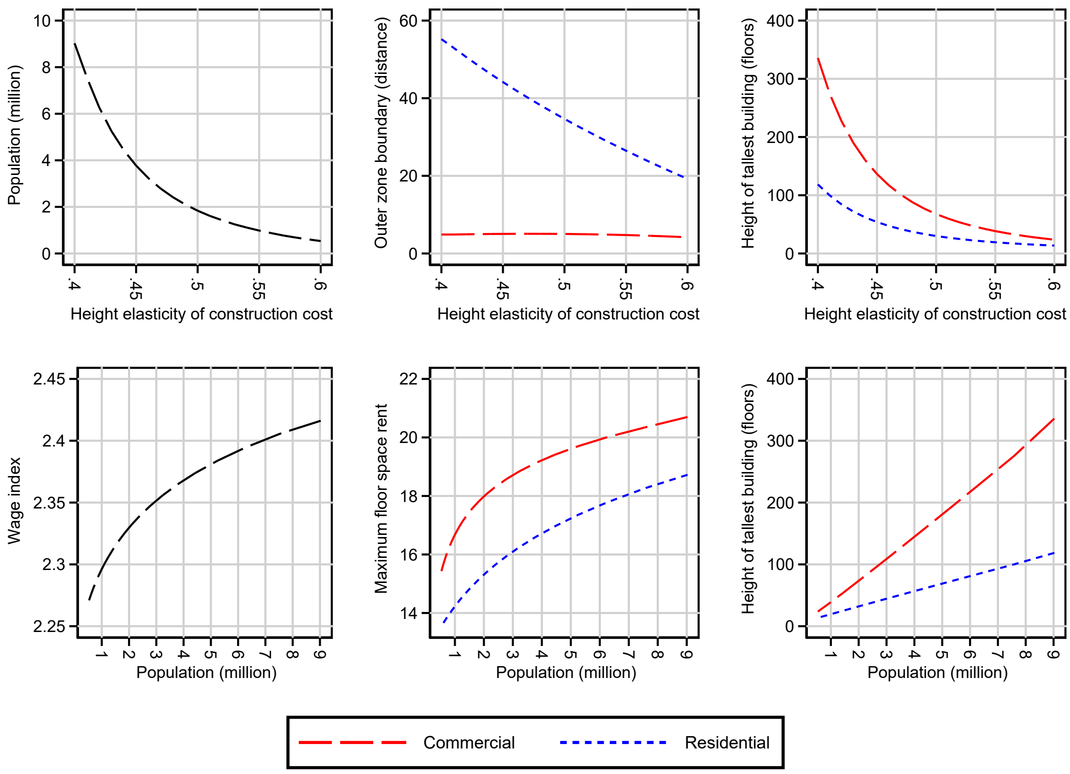


Note: We report solutions to the model developed in Section 3 under varying values of the amenity decay parameters $\{\tau^C, \tau^R\}$. We keep the relative size constant at $\tau^C - \tau^R = 0.005$. All other parameter values are kept constant at the levels reported in Table 1.

construction cost $\{\theta^C, \theta^R\}$. As with the amenity decay in Section 6.2, we select a different value for the commercial height elasticity of construction cost, θ^C, in each run and adjust the residential residential elasticity such that the difference between the commercial and residential elasticity conforms to the baseline, i.e. $\theta^R = \theta^C - 0.005$. This way, we only change one parameter value at a time, which allows us to present the results in Figure 15 using the same structure as in Figures 13 and 14.

A reduction in the cost of height implies that developers will supply more floor space for a given rent level. *Ceteris paribus*, this would reduce equilibrium rents and increase the attractiveness of the city. In our open-city model, population adjusts to restore the exogenous reservation utility. As the population grows, productivity, and wages increase (left panels). To accommodate the larger population, the residential zone expands in horizontal and vertical space. The CBD expands vertically, primarily (upper-middle and upper-right panels). Since profits and utility are anchored to exogenous levels, rents adjust to offset for the increased productivity and wage (bottom-middle panel). Under the chosen parameterization, developers respond to the increased population, productivity and wages by building tall buildings. The city size elasticities of building height, at 0.77 (residential) and 0.94 (commercial), are large. As a result of the rapid expansion of supply of floor space, the city size elasticities of rent, at 0.1 (commercial) and 0.12 (residential), are relatively low (see Appendix Section O.1 for further detail on those estimates). These magnitudes, however, need to be taken with a grain of salt. Our parameterization with a constant height elasticity of construction cost is arguably a reasonable approximation for moderately tall buildings. The elasticity is significantly larger at extreme heights (Ahlfeldt and McMillen, 2018). Without affecting the qualitative implications, we would likely observe a more concave relationship between population and tallest building height if we parametrized the height elasticity of construction cost as a positive function of building height.

Figure 15: Variation in cost of height



Note: We report solutions to the model developed in Section 3 under varying values of the amenity decay parameters $\{\theta^C, \theta^R\}$. We keep the relative size constant at $\theta^C - \theta^R = 0.05$. All other parameter values are kept constant at the levels reported in Table 1.

6.4 Empirical implications

One unambiguous implication that emerges from Figures 13, 14, and 15 is that we should expect a positive relationship between population and the vertical size of cities, theoretically. We subject this model prediction to an empirical test in Figure 16. Indeed, we estimate a positive city size elasticity of tallest building height of 0.25 for a set of international cities with population greater than one million. For smaller cities, there is no significant correlation in the data. We find almost the identical pattern if we consider the second or third tallest building in the city, which suggests that the relationship is not driven by outliers. For large cities, there is also a robust relationship between city size and the number of skyscrapers (150 m+). The implied city size elasticity of the per capita number of skyscrapers is $1.27 - 1 = 0.27$. We estimate a somewhat larger value of 0.34 when restricting the sample to cities in the U.S. This is slightly less than the elasticity of 0.38 estimated by Albouy et al. (2020) for U.S. cities. For China, the estimate is even larger. Again, for cities with a population of less than one million, there is no significant correlation. It appears that there is a size-threshold beyond which the skyscraper technology becomes viable or legally allowed.

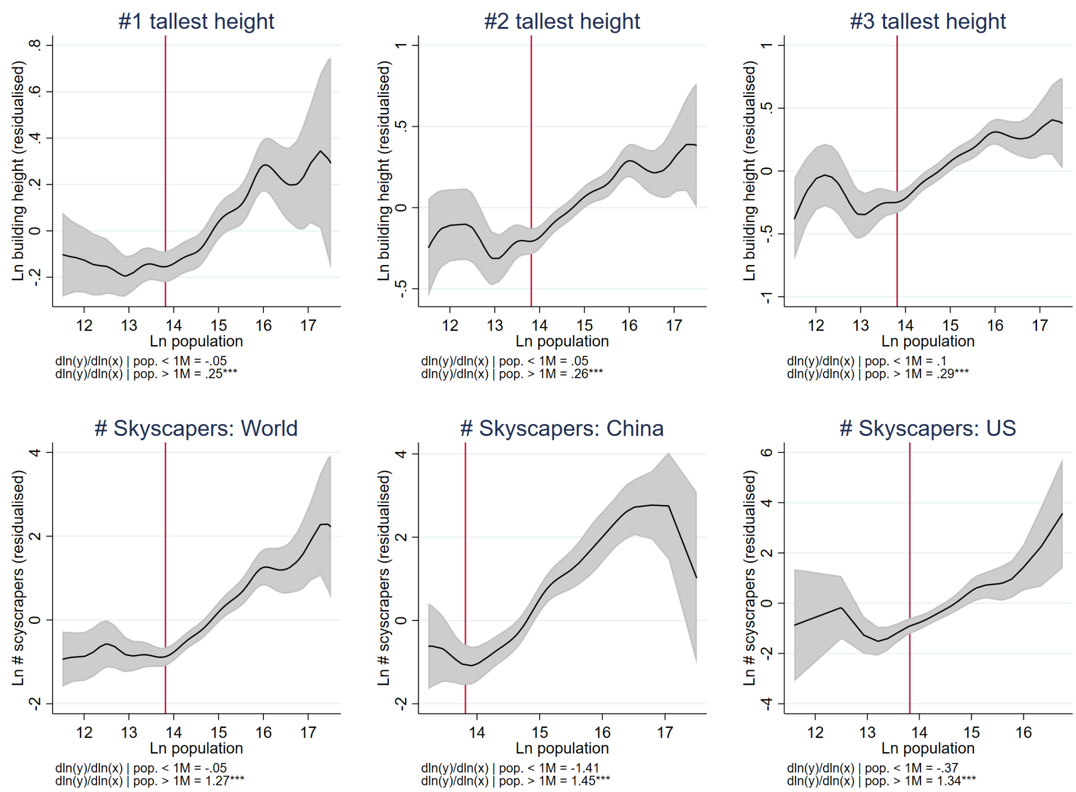
In interpreting the reduced-form relationship between city size and building heights, it is important to acknowledge that we have generated this relationship via three distinct channels in Figures 13, 14, and 15, each of which relates to one major long-run trend. The implication is that, empirically, the magnitude of the city size elasticity of building height will depend on what is driving variation in city size in the data. Given the parameterization of our model, if city size variation comes from transport improvements, we will estimate a lower city size elasticity with respect to building heights as compared to variation coming from the cost of height. Tentatively, we can conclude that the magnitudes estimated from the data suggests that the relationship between city size and building heights observed in our data is likely driven by a mix of both channels.

There are also important implications for the direction of causality. In Figures 13 and 14, we manipulate the demand side of land markets and this causes the supply of tall buildings to respond. Intuitively, urbanization causes skyscraper development. To establish this causal relationship empirically, one would require variation in population that is unrelated to the supply side of land markets. The same logic motivates the use of Bartik (1991) instruments in the estimation the housing supply elasticity (Saiz, 2010). In Figure 15, we manipulate the supply side of land markets and this causes the demand side to respond. Intuitively, skyscraper development causes urbanization. To establish this causal relationship empirically, one would require variation in population that is unrelated to the demand side of land markets. This logic underpins the use of geological conditions as instruments for density in the identification of agglomeration spillovers (Rosenthal and Strange, 2008; Combes et al., 2011), although the role of bedrock as a facilitator of skyscraper development has been challenged (Barr et al., 2011; Barr, 2016).

7 Height regulation

So far, we have treated land markets as competitive and unregulated. Of course, cities impose various forms of zoning and building regulations that affect height decisions (Brueckner and Sridhar, 2012). In this section, we first turn to the welfare implications and how our open-city approach complements recent research that—implicitly or explicitly—refers to the closed-city model (Section 7.1). We next use our model from Section 3 to derive

Figure 16: Skyscrapers and city size



Note: Skyscrapers and population for cities around with world, with at least one 150m+ skyscraper. ln(building height) and ln(# skyscrapers) are residualized in regressions against country fixed effects. The black solid lines are from locally weighted regressions using a Gaussian kernel and a bandwidth of 0.25. Confidence bands are at the 95% level. Vertical line marks the log of one million. Skyscraper data is from https://www.skyscrapercenter.com/; accessed Feb. 2020. Population data are from several sources (available upon request) and are the most currently available counts for the metropolitan regions, which includes the central city and surrounding population agglomerations.

how height limits change the vertical and horizontal structure of a city (Section 7.2).[26]

7.1 Welfare implications

In our open-city model, utility and profits are anchored to exogenous reservation levels. One city's gain from mobile workers is another city's loss. Hence, we focus on the immobile factor, land, to evaluate welfare. The bottom-right panel of Figure 17 illustrates how the aggregate land rent over all uses (commercial, residential, agricultural) within the horizontal area of the unregulated city ($x \in (-x_1, x_1)$) changes as the city introduces a height limit. For a more intuitive interpretation, we normalize the change in land rent by the GDP ($g = \frac{yN}{\alpha^C}$) of the unregulated city.

To lend some further intuition to the counterfactuals, we note that our simulated city

[26] For a broader review of the related literature, as well as a discussion of potential for future research, we refer to Appendix Section P.

shares similarities with Houston, TX. Houston has a population of about two million, the tallest building (JPMorgan Chase Tower) has 75 floors, and the city is famous for being the only major city in the U.S. without formal zoning. Introducing a 10-floor height cap into such a city would reduce the aggregate land rent to the equivalent of 5.8% of the city's GDP. Given a 2012 GDP in Houston of about $400 billion, we can monetize the effect of a hypothetical height cap on annual land rent at $23 billion. Given a 2012 population of Houston of 2.1 million, this corresponds to about $11 thousand per capita and year. At a 5%-capitalization rate, the effect on land value would exceed $450 billion in total and $210 thousand in per-capita terms (see Appendix Section P.3).

The above thought experiment suggests that height caps can have very substantial welfare consequences. In our open-city model, we measure the incidence on the immobile factor: land. The alternative is to use a closed-city model in which workers are immobile and regulation leads to a reduction in worker welfare owing to greater commuting and housing cost.[27] Indeed, the public debate is often more concerned about the incidence of housing supply constraints on workers than on landlords. Also, much much of the economics literature concerned with the measurement of the stringency of height regulations alludes to the closed-city case to derive predictions for housing affordability (Glaeser et al., 2005; Brueckner et al., 2017; Brueckner and Singh, 2020; Jedwab et al., 2020).

Whether to use the open-city or closed-city framework to study the effects of a height limit is a debatable issue. While evidence points to significant migration costs (Koşar et al., 2021), perfect mobility is a commonly accepted assumption of the canonical spatial equilibrium framework (Roback, 1982). Even if workers are imperfectly mobile, one takeaway from Figure 17 is that allowing for taller buildings in an expensive city may be an imperfect solution to an affordability problem since it might only lead to more in-migration, higher wages, and possibly higher rents.[28] Hence, a relaxation of height restrictions will lead to a larger improvement of housing affordability if it is applied to more (if not all) cities within the relevant spatial system.

While the distortionary effects of height caps have been the primary concern of the related economics literature, it is important to acknowledge that height restrictions are implemented for a reason. Planners typically refer to a host of negative externalities associated with tall buildings, such as congestion or shadowing (see Appendix Section P.1).[29] Of course, the optimal building height will be smaller than the profit-maximizing height in the presence of negative externalities, rendering welfare effects of a height limit as theoretically ambiguous. A tentative conclusion from the counterfactuals in Figure 17

[27] Bertaud and Brueckner (2005) formally derive the intuitive predictions that binding height limits force cities to expand horizontally if the city population is fixed. Exploiting height limits in Indian cities, Brueckner and Sridhar (2012) empirically confirm the central prediction that there is a negative relationship between the height limit and the geographic area of city. Their estimates imply that relaxing height limits would increase welfare substantially.

[28] With skill-biased returns to agglomeration, the distributional effects could be regressive (Ahlfeldt and Pietrostefani, 2019).

[29] Indeed, some cities such as London differentiate with respect to the reputation of the architect when enforcing height limits to avoid negative visual externalities (Cheshire and Dericks, 2020).

and the literature reviewed in Appendix Section P.4 is that these negative externalities will have to be large to justify relatively strict height caps.[30]

7.2 Effects on horizontal and vertical structure

Our model developed in Section 3 allows for use-specific height limits, $\bar{S}^U$, which can be set by a planner. As per Eq. (6), these height limits will be binding if $\bar{S}^U < S^{*U}$, where S^{*U} is the profit maximizing building height that we have evaluated thus far. To evaluate the effects of a height limit on urban structure, we solve our model for varying values of a height limit, $\bar{S} = \bar{S}^C = \bar{S}^R$, which we apply uniformly to commercial and residential buildings for simplicity. We correlate a range of endogenous model outcomes with the set values of $\bar{S}$ and in Figure 17.

There is ample evidence that real-world height limits tend to be binding (see Appendix Section P.2). We know from Figure 10 that the tallest building in our unregulated stylized city is slightly smaller than 70 floors. Hence, consistent with reality, only very generous height caps do not affect the spatial structure of the city. An interesting insight from Figure 10 is that, although the tallest residential building in the unregulated city has 30 floors, even height limits greater than 30 floors have an indirect effect on residential building height. This is because unless the height limit reaches the greatest profit-maximizing commercial building height, it constrains the commercial sector, and reduces labor demand, wages, population, and, eventually, demand for residential floor space.

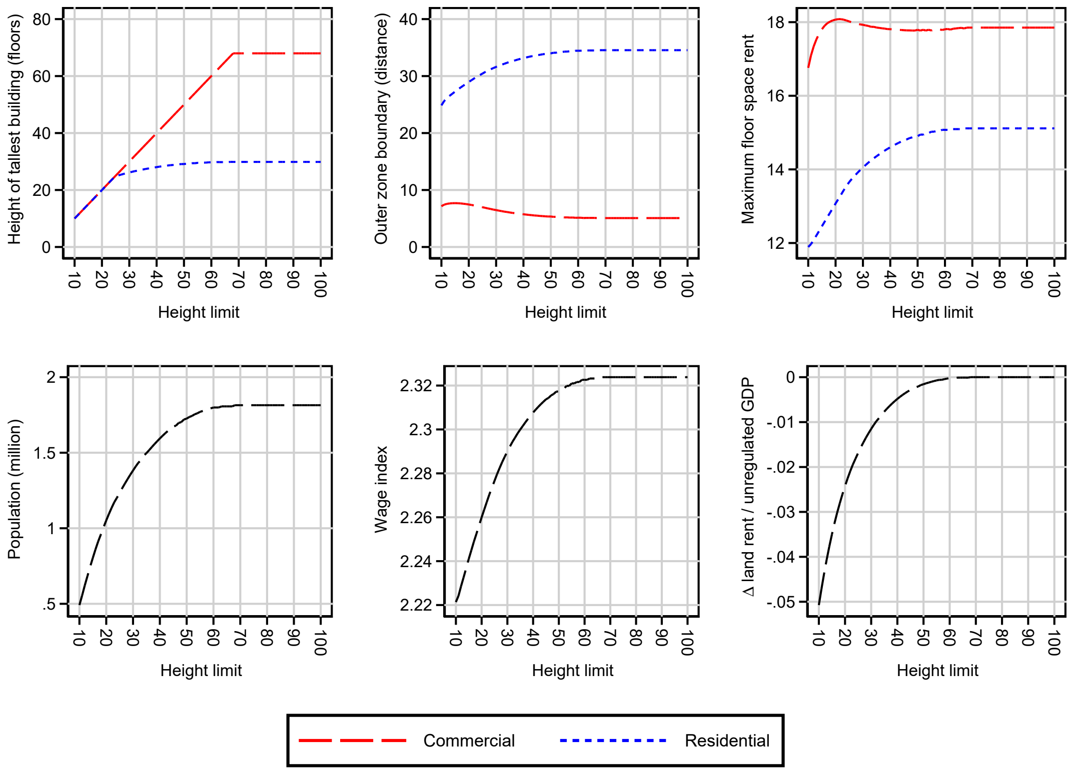
Height limits also have an effect on the horizontal land use pattern. A height limit reduces the amount of commercial floor space at every location where it binds. The limit to vertical expansion results in the CBD growing horizontally, pushing the land use boundary and residential use outwards. The effect on the horizontal size of the residential zone is the opposite. This is because the reduction in the productivity of the commercial sector due to displacement to less productive locations leads to lower wages and a loss of population. The resulting loss of agglomeration economies reinforces this trend. Eventually, there is less demand for residential floor space so that a smaller horizontal zone is required to accommodate the population even conditional on a reduction in vertical space.

The effect of the height limit on residential rent is seemingly at odds with the intuition that supply constraints should increase housing cost. Yet, wages and rents in a frictionless open-city model must adjust to maintain the reservation utility level. If the height limit lowers the wage, the city must lose population until rents have fallen sufficiently to offset for the wage effect. For commercial rents, the effect of the height limit is more nuanced. For intermediate height limits, the supply effect of a more generous height limit offsets for the productivity effect, and commercial rents fall as the permittable height increases. It is the opposite for very stringent height limits. The loss of productivity along with the expansion in horizontal space, facilitated by lower competition for space from fewer resi-

[30] In horizontal settings, evidence suggests that the net effect of land use regulation is negative (Cheshire and Sheppard, 2002; Turner et al., 2014).

dents, dominates the reduction in available vertical space, and rents fall as the permittable height decreases.

Figure 17: Variation in height limit



Note: We report solutions to the model developed in Section 3 under varying values of the height limit $\bar{S}^R = \bar{S}^C$. All other parameter values are kept constant at the levels reported in Table 1. In the bottom-left corner, we measure aggregate land rent over the area occupied by the unregulated city. We do not vary this interval across simulation runs and aggregate over the maximum land bid rent $\max(r^C(x), r^R(x), r^a(x))$ at each location. GDP is the wage bill (yN) weighted by the inverse of the labor factor share α^C.

8 Conclusion

One high-level conclusion of our synthesis is that the literature engaging with the economics of tall buildings is still at an early stage. We discuss the potential future research throughout Appendix Sections J to P. Here, we offer an admittedly subjective selection of three priority areas for research into the vertical dimension of cities.

First, there is a long way to go in understanding how the positive and negative externalities associated with tall buildings play out on balance. Skyscrapers may not just accommodate productive workers in productive locations but may also facilitate gains in productivity, ceteris paribus, if the vertical cost of interaction is sufficiently low. If well-designed skyscrapers put neighbourhoods or cities on the map, they may attract businesses and tourism. At this same time, skyscrapers may generate external costs in the form of shadows, congestion, or the loss of a coherent historic fabric.

Second, there is scope for exploring the costs of and returns to height more fully. The costs of and returns to height appear to be non-linear, and there are likely threshold effects that could be explored with richer data sets. Heterogeneity in the cost of height across uses and the value of the height amenities across users has implications for the horizontal pattern of land use and sorting are, thus, worth being incorporated into quantitative urban models. The net cost of height is one of the main congestive forces and a better understanding of how it evolves over time is helpful with respect to rationalizing changes in the spatial structure in the past as well as anticipating the evolution of cities in the future.

Third, there are forces outside the canonical competitive equilibrium framework that shape the urban height profile and deserve more attention. As an example, disentangling the effects of height competition from fundamentals that would justify extreme building heights remains an empirical challenge. Progress on this front will be essential for economically rationalizing skyscrapers such as the Empire State Building or the Burj Khalifa, which dominate height rankings for decades and exemplify the frontier of technological innovation and human ambition.

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ONLINE APPENDIX NOT FOR PUBLICATION

This section presents an online appendix containing complementary material not intended for publication. It does not replace the reading of the main paper.

I Data

For our empirical work, we collect rich micro-geographic data that fall in four categories. First, we investigate data on tall buildings such as heights and construction costs covering the entire planet that are available from commercial and non-for-profit data providers. Second, we use real estate prices for selected cities such as New York or Chicago spanning more than a century that we have compiled in previous work with different co-authors. We also draw on new data on commercial rents for a wider set of global cities. Third, we look at hand-collected data such as on gross and net floor areas for a global sample of buildings and cash flows for the Empire State Building throughout the 20$^{\text{th}}$ century. Fourth, we analyze global cross-sections of population, GDP and national macroeconomic time-series that are more readily accessible. We summarize the data sources below.

Building heights and related variables. Two databases, Emporis, `Empoirs.com` and the Skyscraper Center, `https://www.skyscrapercenter.com/` were used for skyscraper analysis. Theses databases contain information on building heights, locations, and ages. Each database provides additional information for some buildings, such as gross building area, and construction costs. For annual times series data, from the Skyscraper Center, we used the number of completions per year and heights of tallest building completed each year.

We also use used databases provided by New York and Chicago that give information about every current building in each of these two cities, respectively. One New York City file is from the Dept. of City Planning, which provides building and lot information about every tax lot (called the PLUTO file). We also use data from the so-called building footprint files, we can be found from each city's data source websites. This give height and year of construction for nearly all buildings in the respective cities.

Central business districts. In case studies, we assume for New York City that the center of the CBD is the location of the Empire State Building. For Chicago, the center is the intersection of West Randolph and North State Streets. When we pool mulitple cities, we use global prime locations by metropolitan area as identified by Ahlfeldt et al. (2020) as a measure of the centers of CBDs.

Empire State Building income and costs. Data from the Empire State Building comes from the following sources. Construction costs and land values are reported in Willis and Friedman (1998) and Tauranac (1997). Data on costs and revenues were culled from the archived files of Pierre S. du Pont, one of the major investors in the building. His papers are located in the Hagley Museum Archives, located in Wilmington DE. His files contain annual reports and financial statements up to 1938. Also, the archived files of John J. Raskob and his heirs contain additional data. Files from his heirs contain revenue and costs data in various years in the 1930s and 1940s until the sale of the building in 1951, following Raskob's death in 1950. 1962 net income is reported in the *New York Times* (Aug. 17, 1962). For the period of 1991 to the present, financial and income statements are reported in the SEC 10K forms.

Construction costs. Construction costs for particular buildings are provided by Emporis.

Gross and net floor areas. Gross and net floor areas were culled from the following sources: Sev and Ozgen (2009), Watts et al. (2007), Kim (2004), and Berger (1967).

Land prices. Chicago: Historical land prices are from Hoyt (1933), *Olcott's Land Values Blue Books of Chicago*, and vacant land sales as discussed and used by Ahlfeldt and McMillen (2018). New York: 1950-present are vacant land sales, as discussed and used in Barr et al. (2018). From 1879-1900, data are vacant land sales collected and generously provided by Fred Smith, Davidson College. Data from 1905, 1913, 1921, and 1929 are from Spengler (1930).

Population and GDP. Country populations and gross domestic products are from the World Bank databank, `https://databank.worldbank.org/home.aspx`. City level populations were aquired through various reports and websites, which are available upon request. U.S. GDP time series is from `https://www.measuringworth.com/`.

Rents and Sales Prices. New York: Commercial rent data was generously provided by Cushman & Wakefield. Residential sales price data was generously provided by Streateasy.com. Chicago: residential sales prices was collected from the Redfin.com website. Global sample: commercial rents for office buildings owned by real estate investment trusts (per m^2) from SNL-S&P.

J Stylized facts

This section complements Section 2 in the main paper by providing some background on skyscraper technology and further stylized evidence on the spatiotemporal diffusion of skyscrapers.

J.1 A brief history of the skyscraper

In this section, we briefly review the technological history of skyscrapers and the innovations that have paved the way for cities to increasingly fill the third dimension.

J.1.1 The first skyscrapers

Among architectural historians, there has been a vigorous debate about what constitutes the first true skyscraper. This is because a skyscraper is inherently a multifaceted object, and there is no single definition of what makes a skyscraper a skyscraper. Some point to the first tall commercial structures built after the U.S. Civil War. Others point to the first buildings to use all-steel framing, while others point to the first structures to use their height to convey information about the builders.

Prior to the use of the word "skyscraper" to describe tall buildings, it was frequently used to describe other tall things, including horses, ship masts, and even fly balls in baseball. In the context of buildings, the common usage of the word "skyscraper" in the press pre-dated steel-framing by several years when it was used to describe the class of relatively tall (8-10 floors) commercial buildings going up in New York and Chicago in the early 1880s (Larson and Geraniotis, 1987). Reviewing the suite of technological elements needed to build tall office buildings, engineering historian, Carl Condit, concludes, "If we are tracking down the origins the skyscraper we have certainly reached the seminal stage in New York and Chicago around the year 1870" (Condit, 1988, p. 22).

Nonetheless, the popular belief is that the Home Insurance Building in Chicago, designed by William Le Baron Jenney, and completed in 1885, was the first skyscraper in the world (Shultz and Simmons, 1959; Douglas, 2004). This belief is now widely considered among architectural and engineering historians to be false. The reason is that there does not exist any structure, let alone the Home Insurance Building, that by itself was fundamentally different from the buildings that preceded it and which generated a radical transformation after it.

The widespread thinking is that Jenney invented the steel frame. But this is not true. Rather, his innovation was to embed iron columns and beams inside the two street-facing masonry facades, to make the facades lighter. The two rear walls were standard load-bearing brick. As a result, his building was a hybrid; but it was fundamentally based on the concept of load-bearing masonry (Larson and Geraniotis, 1987). In this sense, Jenny's building represented one of several that were transitional from the traditional wall-load-bearing building to a steel-framed one. Jenney's building, however, was the first to include steel beams (but not columns) for structural purpose's; but again, it was not, by any means, a steel skeleton structure.

When Jenney's building was completed in 1885, there was no mention in the public or academic press about his structure being a new building type. To the engineering community, it seemed one of many that added some innovation to make buildings lighter, allow for more windows, or improve fire safety. His structure was innovative, to be sure, but, at the same time, there is nothing so special about it as to make it qualify as the first clear and true skyscraper.

The idea that Jenny's building was the first began to emerge in 1896, based on a flurry of letters written by Jenney's colleagues to the *Engineering Record*. They essentially "voted" him the winner by popularity contest and not by any rigorous engineering, economic, or aesthetic standards. When Jenney died in 1907, virtually all his obituaries declared him the skyscraper's inventor. In short, Jenney's "victory" was due more to the structure of his social networks rather than his building's structure.

But one thing is for certain, by the early 1890s, the key innovations—the wind-braced, steel-framed skeletal structure and the electric elevator—were in place to remove the technological barriers to height. So that from that time forward, skyscraper height decisions were based on balancing the costs with the revenues and were not so much determined by engineering barriers per se.

J.1.2 The 20$^{\text{th}}$ Century

For most of the 20$^{\text{th}}$ century, technological innovations were incremental. While engineers learned much more about the physics of stabilizing their buildings from wind above and from the geology below, even as late as the 1950s, skyscrapers were still steel-framed boxes. However, improvements in glass technology, fluorescent lighting, and air conditioning allowed for a higher fraction of facades to be covered with glass.

In the 1960s, engineers devised new structural techniques that allowed buildings to go taller without requiring as much steel per cubic meter. The theories behind these ideas were well-known in the engineering profession, but it wasn't until mainframe computing came along and allowed for simulating and testing these ideas to validate them for practical use (Baker, 2001).

As buildings rise higher, the wind forces—the so-called lateral loads—rise exponentially with height. After about 15 stories, stabilizing the lateral loads becomes arguably the dominant element driving increasing marginal costs from adding floors. The most notable innovation of the 1960s was the framed-tube structure. That is, the outer part of the structure is comprised of many closely-spaced columns, which are then attached with horizontal beams. In this way, the building is like a square tube and is sufficiently rigid to prevent significant sway from wind forces.[31] The first building to use this method was the 100-story John Hancock Center (1969) in Chicago. The Twin Towers in New York City used it as well. The Sears (Willis) Tower employed a version of this design by utilizing a tube-within-tube structure.

Besides innovation in structural design and elevators, there have been many other technological improvements over the decades that not only allow for taller buildings, but also improve the quality of occupant life. These include improvements in safety, lighting,

[31] Note that engineers and developers do not aim to make their buildings perfectly rigid, as this would significantly add to the cost of construction. Instead, the goal is to make tall buildings sufficiently rigid so that the rate of sway is undetectable by the human nervous system under nearly all wind conditions.

HVAC, and plumbing and water systems (Condit, 1988; Skyscraper, 2011). More recently, developers are concerned about their building's carbon footprints and aim to use innovations to make them more energy efficient. LEED certification is means to incentivize this process, though the debate is still out on whether new technologies are helping to reduce carbon emissions or are simply "green washing" (Bowers et al., 2020; Scofield, 2013).

J.1.3 Innovations in the 21st Century

Though other structural designs were first used in the 1970s, they have been much more widely employed in the 21st century, especially in Asia (Ali and Moon, 2007). For example, the Burj Khalifa uses a buttressed core, which was first implemented in the 1970s. The building is constructed like a type of pyramid. It has a main central core and three additional wings or cores which buttress the main one. Together, these building cores help to create a much stabler building that can rise 0.83 kilometers with reduced impacts from the wind.

Another innovation is to subject various models to wind-tunnel testing, and the one that most efficiently counters the wind forces is used. For example, Gensler, the architectural firm that designed the Shanghai Tower, subjected different designs to wind-tunnel tests. Based on this, they concluded, "Results yielded a structure and shape that reduced the lateral loads to the tower by 24 percent - with each five percent reduction saving about US$12 million in construction costs"(Xia et al., 2010, p. 13).

Further, many supertall buildings today incorporate mass-tuned dampers. These are large weights hung like a pendulum toward the top of the tower. When the wind forces press against the structure, the damper begins to sway in the opposite diction of the wind, thus dampening the wind's impact (Lago et al., 2018).

Technological innovations in elevators have included ways to use each shaft more efficiently, such as running doubledeckers (one cab on top of another), and eliminating separate machine rooms for raising and lowering the cab cables. Machine-learning algorithms allow cars to more quickly move passengers between floors and reduce waiting times. These innovations not only lower the marginal costs of going taller, but also improve the occupants' quality of life and lower the building's operating expenses (Al-Kodmany, 2015).

J.1.4 The Future

Given the rapid economic growth and urbanization around the world, the demand for supertalls continues to be brisk. Competition in the skyscraper construction industry pushes firms to innovate in order to produce taller buildings at lower costs. One of the key remaining problems is that as buildings become taller, the elevator rope needs to be proportionally longer and heavier. However, at some point, the rope becomes so heavy it can no longer carry itself (Al-Kodmany, 2015). ThyssenKrupp, for example, is developing a rope-less elevator that will move via magnetic levitation. Presumably, once the elevator rope is made obsolete, it will remove a bottleneck to constructing the first mile-high skyscraper.

J.2 Skyscraper completions over time

To quantify the positive long-run trends in heights and volumes, we regressed the log of heights and completions against a yearly time trend in Table A1. Over 120 years, the heights of tallest completions have increased at an average rate of 1.3%. The volume of completions exceeding 150 meters has increased at an even larger percentage of 4.9%. Given that simple log-linear trends explain 63% and 82% of the variation in tallest heights and skyscraper volumes over time, it seems fair to conclude that pronounced vertical growth, historically, has been the norm rather than the exception.

Table A1: Long-run trends in skyscraperization

	(1) Ln(height tallest construction)	(2) Ln(height record-holder)	(3) Ln(# completions)	(4) Ln(# cumulative completions)
Year	0.013***	0.011***	0.049***	0.064***
	(0.00)	(0.00)	(0.00)	(0.00)
Observations	120	120	120	120
R^2	.627	.787	.819	.906

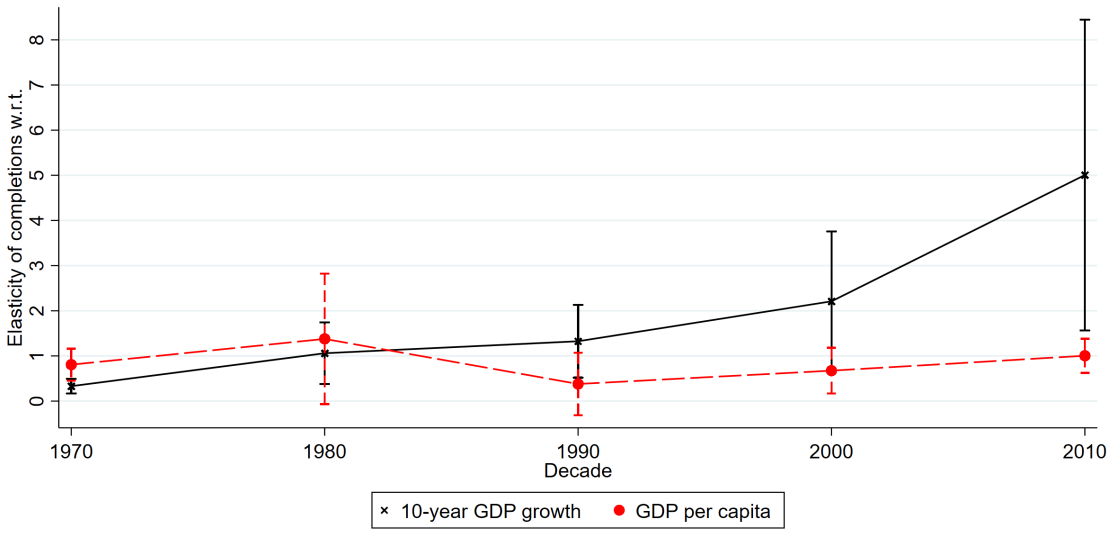
Notes: Unit of observation is years. * p < 0.1, ** p < 0.05, *** p < 0.01

Figure 3 is broadly consistent with the spread of skyscrapers from economically developed countries to countries that are developing. This impression is substantiated by Figure A1. Since the 1970s, economic growth has become a much more powerful predictor of a country's skyscraper completions, whereas the effect of GDP per capita (and population) has remained about the same.

The relationship between economic growth and skyscraper construction is also visible in the time-series of U.S. skyscraper completions depicted in Figure A2. Volumes of skyscraper completions along with tallest heights tend to increase during boom periods. Once the economy contracts, completions and heights plummet. There is a lag of about three to five years in the economic growth effect, which is intuitive since planning and building a skyscraper takes time (panel b). In this context, it is worth noting that there is no evidence for the common belief that skyscraper heights can be used to forecast economic downturns (Barr et al., 2015).

In any case, the sensitivity of vertical growth to the short-run economic cycle is quite striking, given that skyscrapers are among the most durable forms of capital. Emporis, who maintains one of the most comprehensive databases on tall buildings, record only a hand full of teardowns out of nearly 4,000 skyscrapers. The only recorded demoliton or destruction in the class of tall buildings exceeding 250 meters are are New York's Twin Towers in the World Trade Center.

Figure A1: Determinants of skyscraper completions



Note: Markers illustrate estimates from country-level regressions of log # completions against the log of 10-year GDP growth, log of GDP per capita and log of population (not reported for clarity of the graph; the estimated elasticity is close to 0.5 throughout) by decade. Confidence bands are at the 95% level. Sources: Skyscraper data is from `https://www.skyscrapercenter.com/`. Country level population and GDP is from `https://data.worldbank.org/`. GDP is constant 2010 US$.

J.3 Height competition

As discussed in Sections 4.1 and 4.2, average construction costs are positively convex in height and exceeding returns to height at the margin. Thus, at a given time and location, there exists an economic height of the building that maximizes the profits. However, if tall buildings serve non-economic objectives, some buildings may be too tall, in the sense that the chosen height had a marginal cost exceeding the marginal revenue, at the time of completion.[32] The debate about whether skyscrapers, as a class of structures, or particular buildings, are too tall has a long and contentious history, which began in New York City at the end of the 19th-century.[33]

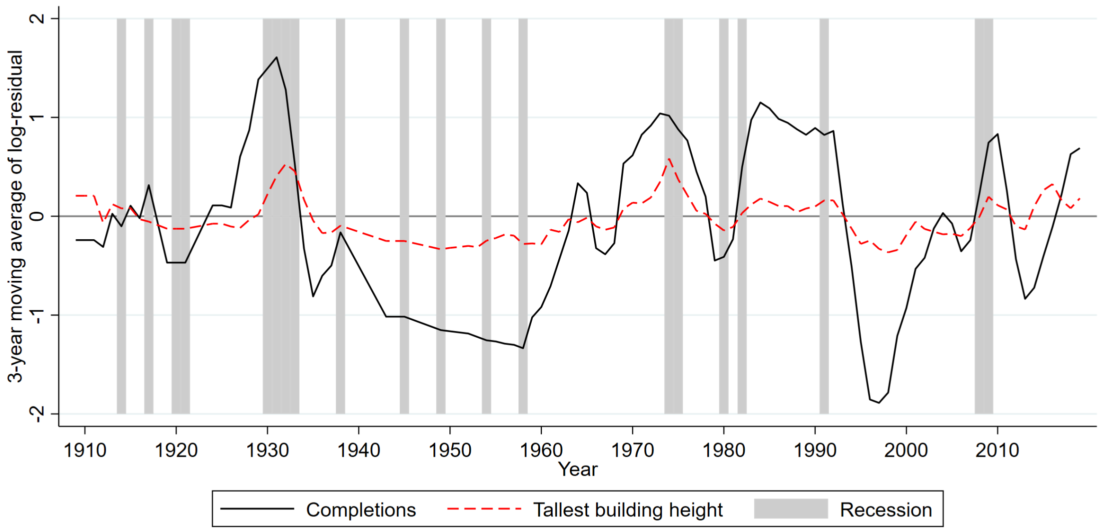
In 1930, Clark and Kingston (1930) wrote their book, *The Skyscraper: Study in the Economic Height of Modern Office Buildings*, to argue that those who called skyscrapers "freak buildings," misunderstood the economics of building tall. They argued that tall buildings were inherently a function of high land values, and they aimed to silence critics who felt that tall buildings were an inefficient use of land.

Nonetheless, they published their book during a famous "height race" in New York City, at the end of the Roaring Twenties (Tauranac, 1997). In 1930, the Bank of Manhattan (283 meters) topped out its world-record-breaking building, only to soon be beaten by

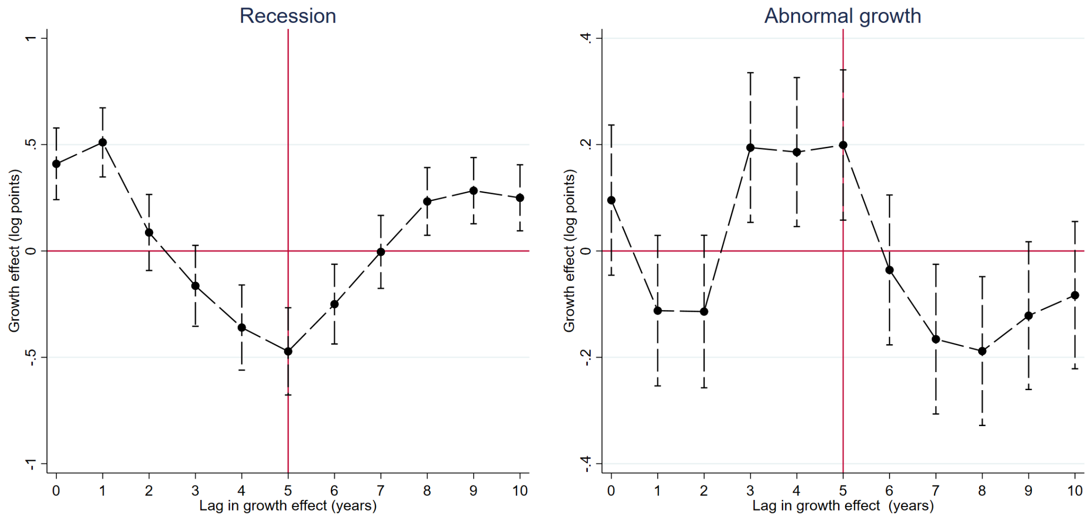
[32] Bercea et al. (2005) argue that in the 16th century, the Roman Catholic church used cathedrals to signal their power when faced with competition from Protestantism.

[33] Note that in 1893, Chicago began a half-century of capping it building heights, thus disqualifying itself from the debate about whether its buildings were too tall or not.

Figure A2: Skyscraper completions in the U.S.



(a) Time trend



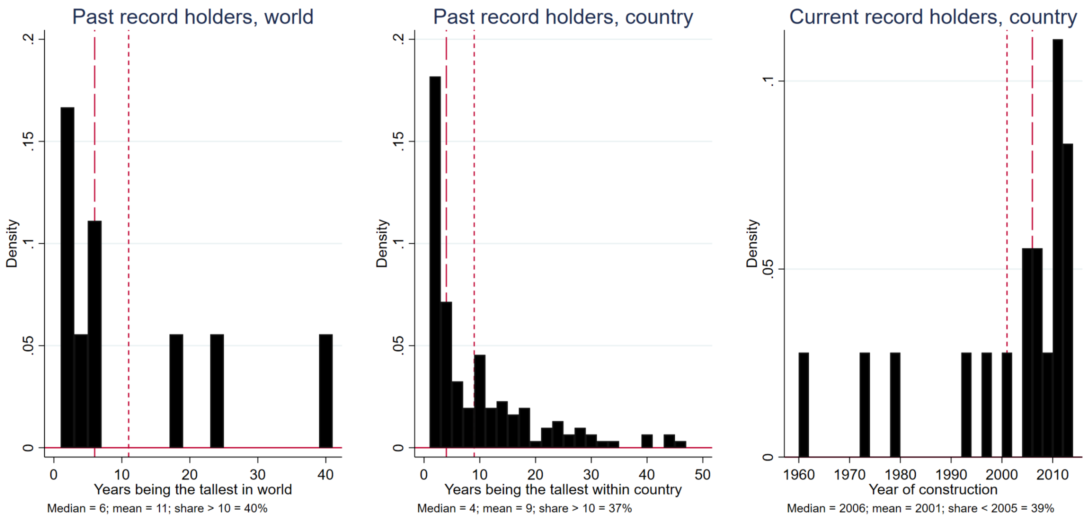
(b) Lag in growth effect on completions

Note: In panel (a), completions and tallest building heights are residualised in regressions against semi-log-linear trends. The residuals are then smoothed using exponential three-year moving averages. Recessions are defined as years with negative real GDP per capita growth. In panel (b), point estimates and 95% confidence intervals are from Poisson regressions of the number of completions against indicators for recessions (left) or abnormal growth (right) lagged by the indicated number of years, controlling for a time trend. Recessions are defined as years with negative real GDP per capita growth. Abnormal growth periods are periods with the real GDP per capita growth exceeds the long-run median (about 2%). Skyscrapers: https://www.skyscrapercenter.com/. Real GDP per capita: https://www.measuringworth.com/

the Chrysler Building (318 meters), which within a year was beaten by the Empire State Building (381 meters) in 1931. This race to dominate the skyline has only fueled the perception that ego-driven developers are imposing their will on the skyline, and the economics be damned (Barr, 2016).

There is also the perception that developers of the world's tallest building add extra-height to preempt would-be entrants, thus allowing the record holders to keep their records for extra time. Supporters of this theory can point to the Empire State Building, which held its record for 40 years. However, there are also cases of rapid-fire succession. 40 Wall Street held its record for a matter of months. The Twin Towers lost their record within a year. And the Petronas Towers (1999) (only a record due to its spires, not its floor count) lost its record to the Taipei 101 within five years. Figure A3 shows that a long duration at the top of the height ranking is the exception rather than the rule. The median record-breaking building holds its position for six years. Within countries, the median duration is four years. Across countries, the median year of completion of the tallest building is 2006. This relatively fast succession suggests that long-run trends in economic fundamentals are an important driver vertical growth.

Figure A3: Years being the tallest building



Note: Record holders are buildings holding the title of the tallest building in the world or within their country. Dashed vertical line mark the median. Dotted vertical line marks the mean. Data as of 2015 from Emporis, previously used by Ahlfeldt and McMillen (2018).

More broadly, the ability to use case studies to generalize about the motivations of record-breaking developers is limited. The reasons for building "too tall" are multifaceted. Cities in Asia have more complex land markets and greater government involvement in supertall projects. Skyscrapers may have additional second-order economic benefits that can be confused for or correlated with "ego-based" height. This can include various types of advertising or signaling, and which may come from the builder, the occupants, the city, or the nation (Watts et al., 2007; Garza, 2017). Developers may also use their structures as "loss leaders" to increase land values of surrounding properties and to draw income from tourism. Observation decks in the clouds are profitable ventures that can generate as much as one third of total income, as in the case of the Empire State Building.

J.4 A case study on a "too tall" Building

In this section, we evaluate the profitability of the Empire State Building (ESB) in a case study.

J.5 The Empire State Building

The ESB is a 102-floor skyscraper in Midtown Manhattan, New York. The Art Deco building was completed in 1931. With a roof height of 380 meters (443 including the antenna), it was the tallest building in the world for about four decades, longer than any other building ever since. The building cost $40,948,900 to build ($16 million was the market price for the lot), including demolition of the Waldorf Astoria Hotel which previously occupied the site of 8,487 square meters (Willis and Friedman, 1998; Tauranac, 1997). This is equivalent to $554,644,100 in 2018, which is roughly the amount that was invested during a major renovation project in 2009. The building has become famous for its height and iconic design and infamous for its allegedly poor economics (Kingwell, 2006). There seems to be some consensus that its extraordinary height was driven by some value of being the tallest (Tauranac, 1997; Helsley and Strange, 2008).

J.5.1 Ex-ante analysis

Before we turn to the evaluation of the realized returns on investment, we summarize the business case from the perspective of the developers when they made the initial height decision. Table A2 shows the ex-ante analyses of the Empire State Building along with the hypothetical development in (Clark and Kingston, 1930, henceforth CK), as the two projects were similar in scope. For this analysis—to compare apples to apples—we look at expected return on investment (ROI), which is calculated as expected net income with 100% occupancy divided by the total cost for the structure (land costs and carrying charges, and hard and soft construction costs). The ROI analysis does not include the cost of financing.

The CK lot was across the street from Grand Central Station. The ESB lot was 0.8 km south of that. The ESB project was originally costed for an 80-story building (but later on, more height was added). The CK building is for 63-stories. Total project costs are similar ± $40 million. CK estimated average rents of $ 3.80 per square foot. The ESB rents were estimated to be lower at $3.36, which seems reasonable given its location.

The "ideal" ROI from CK was 10.24%. Even though the ex-ante estimates for the ESB were lower, they were still reasonable and relatively high, at 9.54%. In short, despite the legend of the Empire State Building being a foolhardy idea, the ex-ante returns showed it to be a reasonable project for the time and, especially given the fact that nobody could have foreseen the severity of the Great Depression that was soon to follow.

Table A2: Ex-ante case study of Empire State Building

	ESB	CK
	Building Specs.	
Lot size (sq. ft.)	$91,351	$81,000
Gross Builidng Area	$2,812,739	$2,444,212
Net Rentable Area	$2,080,000	$1,653,342
Floors	80	63
	Costs	
Total Land Cost	$16,000,000	$16,200,000
Total Building Cost	$24,718,000	$19,390,000
Land Carrying Charges	$4,063,791	$3,552,000
Total Project Cost	$44,781,791	$39,142,000
	Income	
Total Office Revenue	$7,000,000	$6,302,000
Operating Costs	$2,730,000	$2,292,000
	Profitability	
Net Operating Income	$4,270,000	$4,010,000
Return on Investment	9.54%	10.24%

Note: ESB gives projected numbers for the Empire State Building in 1929. Source: Barr and Ahlfeldt (2020). CK is analysis for a hypothetical building from Clark and Kingston (1930).

J.5.2 Ex-post analysis

One might be concerned that the developers purposely exaggerated their expectations to make the business case look more positive. To shed light on the return on investment the ESB actually generated over its first lifetime, we now conduct an ex-post case study of discounted cash flows based on observed data. We assume that the developer uses 100% equity to pay for the cost of land acquisition and construction. After 78 years, the building has fully depreciated, the stream of net operating income (NOI) ends, and the developer sells the lot. This is consistent with the major renovation in 2009 that effectively marks the beginning of the second lifetime of the building. As a measure of long-run profitability, we compute the net realised return (NRR), which adjusts the realised returns for the risk-free rate.

Data. Historic income and costs data from the 1930s and 1940s are from annual and quarterly financial statements from the du Point and Raskob files at the Hagley museum archives. More recent data points are from SEC 10-K forms for the building (or the REIT that owns it). Additional data points came from historical New York Times. See Table A3 for details. We use nearby vacant land sales to approximate the value of the value of the lot in 2008 (Barr et al., 2018). The risk-free rate is the U.S. Short Term Ordinary Contemporary (https://www.measuringworth.com). The long-run time series of the S&P composite stock market index and dividend yield is from Williamson (2020).

Net Operating Income. As the net operating income (NOI), we consider the building's total revenues, including the lease of floor space and the operation of the antenna and the observatory deck, net of all operation costs. Costs include property taxes, but exclude interest, amortization, and depreciation since we assume the buildings is paid 100% out of equity and the capital value is zero at the end of the lifetime. Our NOI directly corresponds to the conventional definition of earnings before interest, taxes, depreciation, and amortization (EBITDA). Since we do not observe the NOI in all years, our first task is to interpolate a time series based on the the data points we observe. To this end, we employ the following spline regression model:

$$\ln NOI_t = b_0 + b_1(t - O) + b_2[(t - K) \times I(t < K)_t] + e_t,$$

where t indexes years, $O = 1931$ is a scalar representing the opening year and $K = 1954$ is a scalar representing the first time when the ESB was transacted. The spline equation allows for a change in slope, but not level in this year (a "knot"). b_0, b_1, b_2 are parameters to be estimated and e is the residual term capturing the effects of events that lead to deviations from the long-run trend. In the remainder of our DCF analysis, we define the *factual* long-run trend in NOI as:

$$\widehat{NOI_t} = \exp\left(\hat{b}_0 + \hat{b}_1(t - O) + \hat{b}_2[(t - K) \times I(t < K)_t]\right).$$

In the upper-left panel for Figure A4, $\widehat{NOI_t}$ is the dashed line. NOI during the 1930s and 1940s was likely negatively affected by the Great Depression and World War II. To approximate a *counterfactual* scenario without the adverse economics effects, we backwards extrapolate the factual NOI trend observed since 1954. Hence, the counterfactual NOI is

$$\widetilde{NOI_t} = \exp(\hat{b_1}(t - O) + \hat{b}_0).$$

Its departure from the factual NOI trend is maked by the dotted line in the upper-left panel of Figure A4. This dotted line may be taken as a crude approximation of what might have been the expected NOIs, whereas the dashed line represents the historical realisation out of a distribution of possible realisations centred on the dotted line.

Opportunity cost of capital. To discount NOIs to 1930, we use a cumulative discount factor based on the opportunity cost of capital, which we set we set to the risk-free rate r.

$$CDF_t = \frac{1}{\prod_{s=1930}^{t}(1 + r_s)}$$

We plot the time series along with the risk-free rate in the upper-right panel of Figure A4. Accordingly, capital cost were low until the 1950s and then increased up until the the early 1980s when the trend reverses.

Discounted cash flow. The discounted cash flow (DCF) observed in t and valued in 1930 is then simply the product of the NOI and the CDF:

$$DCF_t = \widehat{NOI}_t \times CDF_t$$

The cumulative discounted cash flow up until year t is

$$CDCF_t = \sum_{s=1930}^{t} DCF_s.$$

The bottom-left panel of Figure A4 plots the time series of DCF_t and $CDCF_t$. Evidently, early cash flows suffer from the economic crisis. These early years constitute the basis for the bad reputation the ESB enjoys in terms of profitability. Indeed, Alfred Smith, President of Empire State, Inc., towards the end of the 1930s, asked for a tax discount on the grounds of the building making losses an his company was at risk of bankruptcy.

However, it is important to notice that only during the first half of the 1930s, the total cash income did not exceeded operating expenses and taxes. Earnings before interest, taxes, depreciation, and amortization turned positive in 1938 already. The problem was that the building was financed with a relatively low equity share. Hence, the NOI was still below the borrowing cost, creating a severe liquidity problem for the building's owners. But the building did deliver a positive yield. After the Great Depression, NOI increases rapidly. In 1953, the building laready breaks even in the sense that the cumulative sum of discounted cash flows equates to the total cost (land and construction marked by the solid horizontal line). The DCF peaks in the early 1960s. Later cash flows are large and positive, but increasingly discounted by the increasing opportunity cost of capital.

Net realised return. A risk-averse investor will demand some return above and beyond the risk-free opportunity cost of capital when embarking on a venture as ambitious as building the world's tallest building. We refer to the net return the investor obtains after taking out the risk-free rate as the net realised return R. To find R for the ESB, we combining the above ingredients and search for the value of R that satisfies the condition

$$\underbrace{I_{t=1930}}_{\text{Land and construction cost}} = \underbrace{\sum_{t=1930}^{t=2008} \frac{\overline{NOI}_t}{\prod_{s=1930}^{t}(1+r_s)(1+R)}}_{\text{Discounted cash flows}} + \underbrace{\frac{S_{t=2008}}{\prod_{s=1930}^{s=2008}(1+r_s)(1+R)}}_{\text{Discounted land value}},$$

which just states that the sum of the NOIs over the life time of the building plus the final value of the lot, discounted by the cost of capital and the net realised return, must equate to the initial investment I (land and construction cost). We assume a value of the lot in 2008 of $S_{t=2008}$ = \$400 million. This implies a per-square-foot price of slightly more than \$4,500 which is towards the lower end of nearby vacant land sales we observe in the data. In practice, this choice is not particularly crucial. For non-marginal rate of returns, the effect of the final land value will be negligible owing to the long period over which it is discounted. Given the non-linearity of the equation, we use an iterative procedure to solve for R.

It is unlikely that the developer anticipated the adverse economic performance during the 1930s and 1940s when making the height decision. To evaluate whether the height decisions was grounded in fundamentals it is, therefore, useful to approximate the trajectory of expected NOIs. To this end, we evaluate different scenarios which represent weighted combinations of the factual and counterfactual scenarios illustrated in the upper-left panel of Figure A4.

$$\overline{NOI}_t = w\widehat{NOI}_t + (1-w)\widetilde{NOI}_t$$

With a weight of $w = 1$ we obtain the R for the factual scenario. With $w = 0$, we obtain R for the counterfactual that eliminates the effect of the economic crisis. $0 < w < 1$ naturally give weighted combinations of both scenarios whereas $w > 1$ amplifies the effects of the crisis.

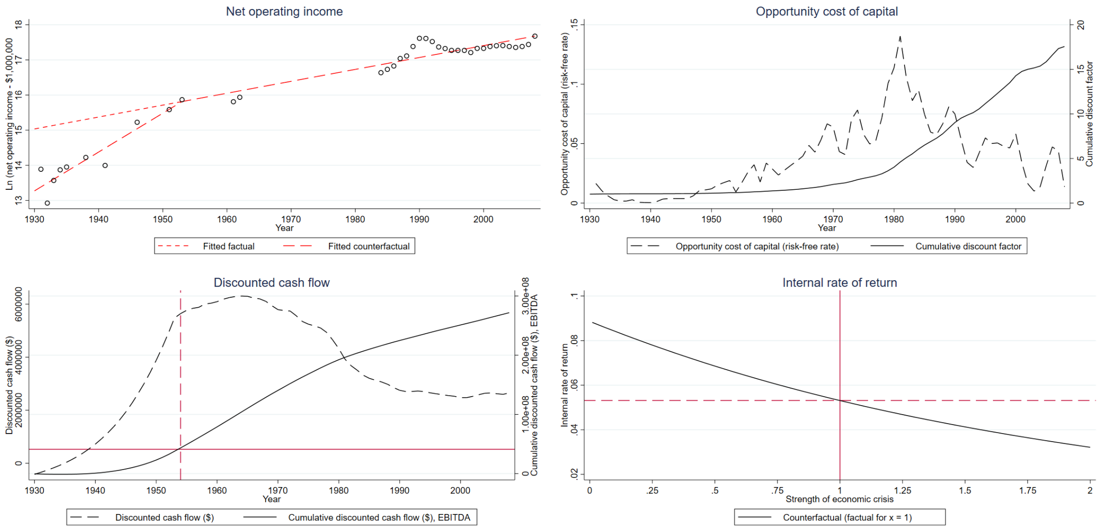
We plot our solutions for R as a function of w in the bottom-right panel of Figure A4. Considering the no-crisis $w = 0$ scenario, we obtain an internal rate of return of about 8% which is very solid. For comparison, CK estimate that in 1929 the following returns on investment could be expected depending on the height of the building: 8 floors: 4.2%; 13 floors: 6.44%; 22 floors: 7.75%; 30 floors: 8.5%; 37 floors: 9.07%; 63 floors: 10.25%; 75 floors: 10.06%. However, our R is net of the opportunity cost of capital whereas CK's ROIs are gross returns. Using the factual NOIs affected by the crisis, our R drops to 5.4%, which is less impressive but still sizable given that our returns are net of of the opportunity cost of capital. Even if we significantly increase the impact of the crisis on the NOI, over its lifetime, the ESB still delivers a positive return above and beyond the opportunity cost of capital.

Comparison to stock market. Another way of putting the low NOI during the 1930s and 1940s into perspective is to compare the cash flow of the ESB to an alternative asset class. A popular benchmark is the stock market. We use the long-run time series of the S&P composite stock market index and dividend yield from Williamson (2020) to compute the return on a stock-market investment that equates to the total cost of the ESB. In the left panel of Figure 5, we plot the discounted NOI of the ESB (from the bottom-left panel of A4) against the discounted dividend payments from an investment into the stock market:

$$\underbrace{DD_t}_{\text{Discounted dividend}} = \underbrace{i_t}_{\text{yield}} \times \underbrace{I_{t=2008}\frac{M_t}{M_{t=2008}}}_{\text{Asset value}} \times \underbrace{\frac{1}{CDF_t}}_{\text{Discount factor}}$$

In the right panel of Figure 5, we plot the discounted asset value which is simply the product of the two last terms in the above equation for the stock market. For the ESB the asset value corresponds to the total cost of structure and land in 1930 and the pure

Figure A4: Case study of Empire State Building



Note: We interpolate annual net operating income in a regression of observed values against a spline function of a yearly trend, allowing for change in slope in 1953, the first time the building was transacted. The cumulative discount factor is the factor that discounts a cash flow in a given year to a present value in 1930, based on the observed risk-free rates up that year. Discounted cash flows are based on the factual predictions of the net operating income, discounted by the cumulative discount factor. The solid horizontal line in the bottom-left panel is the total cost of construction of about $40 million. The dashed vertical line in the same panel indicates the break-even year 1953. The internal rate of returns are excess returns in addition to the risk-free rate. The strengths of the crisis moderates counterfactual net operating incomes from 1930 to 1954. At one, we use the factual predicted values. At zero, we use the counterfactual marked by the dotted line in the upper-left panel. At two, the effect of the crisis (difference between dashed and dotted line in the upper-left panel) is twice as large. The horizontal line gives the factual IRR = 5.4%

land value in 2008 since we assume that the structure capital has fully depreciated.

Evidently, stock market returns also took a hit during the Great Depression, although not quite as hard as the revenues of the ESB. Yet, the recovery is also stronger In ESB's NOI, which exceeds stock market dividend by a large margin during the 1950s, 60s, and 70s. In terms of asset value, the stock market portfolio naturally outperforms the ESB as its structure capital, which accounts for about two thirds if the initial asset value, depreciates to zero. Yet, over the (fist) lifecycle of the ESB, the higher discounted NOI weighs more (as it occurs earlier) than the lower exit value as the stock market's net realized return, at 4.3%, is lower than the 5.4%-return of the ESB.

J.5.3 Summary.

Our ex-post case study of the economics of the ESB confirms anecdotal evidence reporting that the building was not particularly profitable during the 1930s and the early 1940s. Yet, it appears that the poor economic performance was mainly attributable to a severe economic downturn which was hard to foresee. Our best attempt to adjust for the economic cycles delivers a fundamental return that is in line with industry standards. Indeed, the ESB over its (first) lifetime generated returns that exceeded the stock market. Our results

Table A3: Data on net operating income (NOI)

Year	**NOI (\$)**[a]	Source	Notes
1930	-40,948,900	Willis and Friedman (1998)	Total cost of land and construction
1931	75,250	du Point and Raskob files at the Hagley museum	-
1932	-589,539	du Point and Raskob files at the Hagley museum	-
1933	-217087	du Point and Raskob files at the Hagley museum	-
1934	57,680	du Point and Raskob files at the Hagley museum	-
1935	146,215	du Point and Raskob files at the Hagley museum	-
1938	503,966	du Point and Raskob files at the Hagley museum	-
1941	198,901	du Point and Raskob files at the Hagley museum	Costs imputed
1946	3,097,828	du Point and Raskob files at the Hagley museum	Costs imputed
1951	4,875,881	du Point and Raskob files at the Hagley museum	-
1953	6,791,535	NY Times, May 26, 1954, p. 31	-
1961	6,352,694	NY Times, Aug. 17, 1962, p. 33	Based on first 5 months
1962	7,341,948	NY Times, Aug. 17 1962, p. 33	Based on first 5 months
1984 -2008	15,810,000 46,552,067	https://www.sec.gov/Archives/edgar/data/32776/ 000003277609000017/esbcamendmentl.htm	-

Notes: [a] Earnings before interest, taxes, depreciation and armotization (EBITDA) in nominal values.

from the ex-post analyses of discounted cash flows is consistent with the market price the building achieved in 1954. In its first transaction, the building sold at a price of about \$51 million, which is close to and even slightly more than the 1930 total land and construction cost, inflated by the opportunity cost of capital. The implication is that by the mid-1950s, the market expected the building to deliver an average return. In this context, it is worth noting that the building was purchased by Harry Helmsley in 1961. Helmsley built a reputation for developing a highly profitable portfolio that made him real estate billionaire. Put simply, our case study suggests that the ESB has been more profitable than critics have claimed. While the egos of the developers may have made it "too tall" at the time, the desire to make economic profits seemingly ensured that the Empire State was not built "much too tall."

J.6 Related literature

Height competition. Helsley and Strange (2008) offer a game-theoretic model of height competition. Using a smaller sample of cities than in Figure 4, they observe that a city's tallest building is sometimes much higher than the second tallest building. They argue that if two developers are vying to claim the prize of the "tallest building" for bragging rights or personal satisfaction (i.e., ego), then a pure-strategy equilibrium will have one builder constructing a building so tall it will have zero economic profit. In a mixed-strategy equilibrium, each builder will assign a positive probability to building too tall. In a sequential game, developers deliberately build extra tall to deter competitors. In short, they demonstrate that "too tall" construction can be rationalized theoretically.

Prompted by this theoretical contribution, there has been some work on empirically measuring the effects of height competition. However, it has remained a challenge to separate competition effects from the effects of unobserved fundamentals, so the evidence is best interpreted as suggestive.

Barr (2010) studies the economics of skyscraper construction in Manhattan using time series data from 1895 to 2004. He estimates the economic heights and number of completions with a regression that includes economic fundamentals. While he cannot rule out height competition from time to time and for particular buildings, skyscraper heights, on average, are consistent with economic fundamentals.

Barr (2012) analyses the effects of height competition by regressing the heights of a newly-completed buildings on a weighted average of the heights of surrounding structures (completed before or contemporaneously). The estimate for the spatial autoregressive parameter turns out to be positive and statistically significant, but is economically small. Extra height is mostly added during boom times when the forgone profits are relatively more affordable.

Barr (2013) investigates height competition between two rivaling skyscraper cities, namely New York and Chicago. He uses time-series measures of each other city's skyscraper construction (average heights and completions) to estimate skyscraper reaction functions. He finds that each city adds heights in response to the (lag of) building activity in the other city, but, again, the interaction coefficients are economically small.

Barr and Luo (2020) analyze skyscraper construction across Chinese cities. They find that local GDP and population are strong predictors of skyscraper height and completions, which suggests that China's rapid urbanization is driving the rise of its skylines. They also find that cities with younger officials build taller skyscrapers, suggesting the desire to use icons for career promotion in a system where municipal officials control land use through land leases (Brueckner et al., 2017). From spatial autoregressions, they infer that same-tier cities are competing against each other in the height market, in order to advertise their cities and its officials.

Using a different approach that draws from the spatial point-pattern literature (Duranton and Overman, 2005), Ahlfeldt and McMillen (2015) document that the completion of a very tall building is followed by a period of less ambitious constructions in the immediate neighbourhood. While this finding is consistent with the sequential-game predictions of the Helsley and Strange (2008) model, an existing tall building may also lead to a lower economic height for subsequent buildings due to a lower view amenity.

White Elephants. Gjerløw and Knutsen (2019) test the hypothesis that autocratic leaders build "white elephants" as symbols of their reigns, or to demonstrate their power to muster significant resources, or to provide work or provide incomes to their supporters. They show that going from maximum democracy to maximum autocracy is associated with about one extra skyscraper, ceteris paribus. They also find that autocracies are more likely to add more "vanity height" to their structures, where vanity height is the difference between the roof-line and the height of the topmost part of the building. They interpret this extra height as the desire to signal the power of autocratic leaders.

Urban Planning and Growth. Anecdotally, cities world-over appear to be using supertall buildings as catalysts of urban development and renewal. For example, the so-called Three Brothers of Shanghai (the Jin Mao Tower, the Shanghai World Financial Center, and the Shanghai Tower) were part of the master plan for the Lujiazui financial district. The Burj Kalifa and the Jeddah Tower (under-construction) are central features of new neighborhoods being developed. The original Twin Towers in New York were part of a strategy to modernize and re-invigorate lower Manhattan, which had fallen on hard times after World War II. Unlike for sports stadia (Coates, 2007; Ahlfeldt and Kavetsos, 2014), there is no work in economics that explores how skyscrapers induce positive spillovers in terms of land values, house prices, or foreign direct investment.

The skyscraper curse. The so-called Skyscraper Curse alleges that supertall–especially record-breaking–building completions are a herald of economic doom (Lawrence, 1999). Supporters point to the Empire State Building, completed in 1931, and the Burj Khalifa, completed in 2010, to show that these towers were finished during severe downturns. Since there are about twice as many economic crises than record-breaking buildings over the past 100 years, it is relatively easy to "manually" pair record-breaking buildings with business cycle peaks. Barr et al. (2015) are the first to rigorously test the skyscraper curse hypothesis using Granger-causality analysis. The result is that height cannot be used to predict the business cycles, but economic growth can be used to forecast building heights.

J.7 Potential for future research

Each generation redefines its own "normal" building height. In the late 19th century, 15 stories was considered excessive. By the 1920s, 40 stories was not uncommon. Today 100-story structures are regularly built. As the barriers to building height continue to fall away, the debate about whether skyscrapers are too tall keeps renewing itself.

Empirical research into whether the tallest structures in the world are, in fact, economically too tall remains challenging due to data limitations. Thus far, comprehensive data that are readily accessible are confined to completed heights of buildings and the general economic climate in which they are built. Building-level data on revenues and the costs of land acquisition and construction cost will be key to evaluating the economic case for building super-tall.

Since skyscrapers are durable and there are significant construction lags, expectations matter for building height decisions. Seemingly excessive structures may be result of perfectly foreseen economic growth or a myopically extrapolated real estate boom. Understanding the extent to which cyclical vertical growth can be rationalized under perfect foresight may be informative with respect to the role of expectations on real estate markets more generally.

K Model

This section complements Section 3 in the main paper. First, we provide evidence that buildings tend to specialized in particular uses, justifying the uniform within-building land-use assumption in our model. Second, we introduce the numerical procedure to solve for the equilibrium of the model. Third, we derive the elasticity of height with respect to land rent. Finally, we discuss the related theoretical literature and an avenue for future research.

K.1 Within-building land use

In our model, we assume that land use is uniform within buildings. This assumption is motivated by the real-world observation that buildings are strongly specialized on particular land uses. To provide systematic evidence on this stylized fact, we compute the Herfindahl-Hirschman Index of within-building use as follows:

$$HHI_i = \sum_{u}^{\mathcal{U}} \left(\frac{F_{i,u}}{\sum_u^{\mathcal{U}} F_{i,u}} \right)^2,$$

where $F_{i,u}$ is the total floor space of use u in building i. We consider seven land uses, including residential, office, retail, garage, storage, factory, and other observed across more than 810 thousand buildings in New York City.

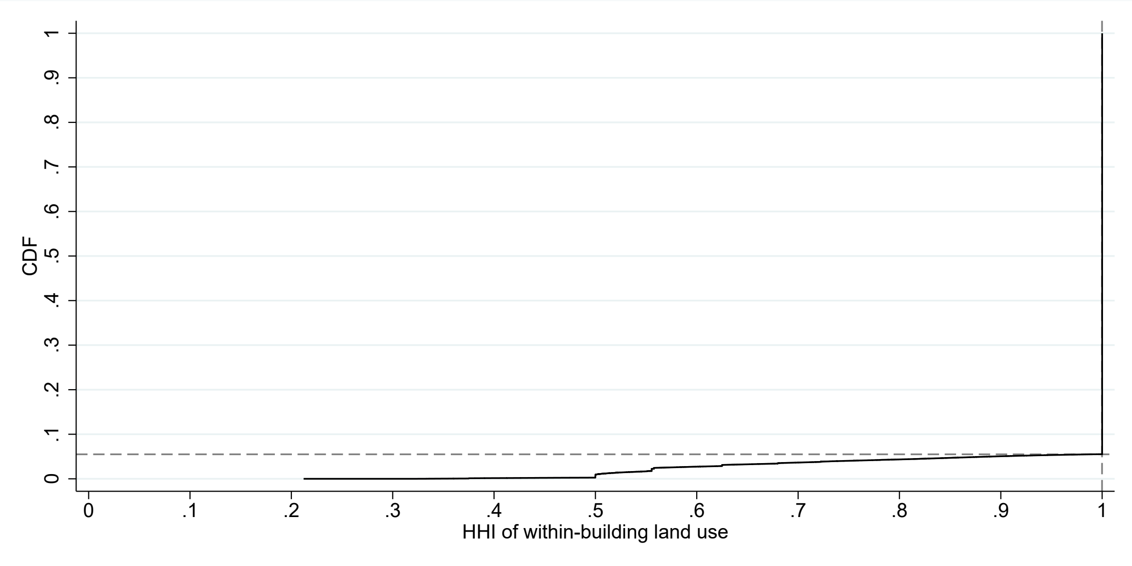
We illustrate the cumulative distribution function (CDF) of HHI_i in Figure A5. Buildings in New York City are highly specialized. In fact, we find an HHI of more than 0.999 for nearly 95% of buildings in New York City. To allow for some spatial disaggregation, we present descriptive statistics of the distribution of HHI_i across buildings by borough in Table A4. Even in the most vertical borough, Manhattan, the degree of building specialization is very high, although somewhat lower than in the flatter boroughs.

Table A4: HHI of within-building use by borough

Borough	Mean	Std. Dev.	Freq.
Manhattan	0.872	0.176	40,072
Brooklyn	0.955	0.136	263,866
Queens	0.976	0.104	309,948
Bronx	0.977	0.095	82,598
Staten Island	0.991	0.065	114,247
Total	0.966	0.117	810,731

Note: We compute Herfindahl-Hirschman Index (HHI) using the shares of different uses at total floor space for 810,731 buildings in New York City. Source: 2017 NYC PLUTO file from the NYC Dept. of City Planning. Note that data is for virtually all buildings in New York City.

Figure A5: Specialization of use within buildings



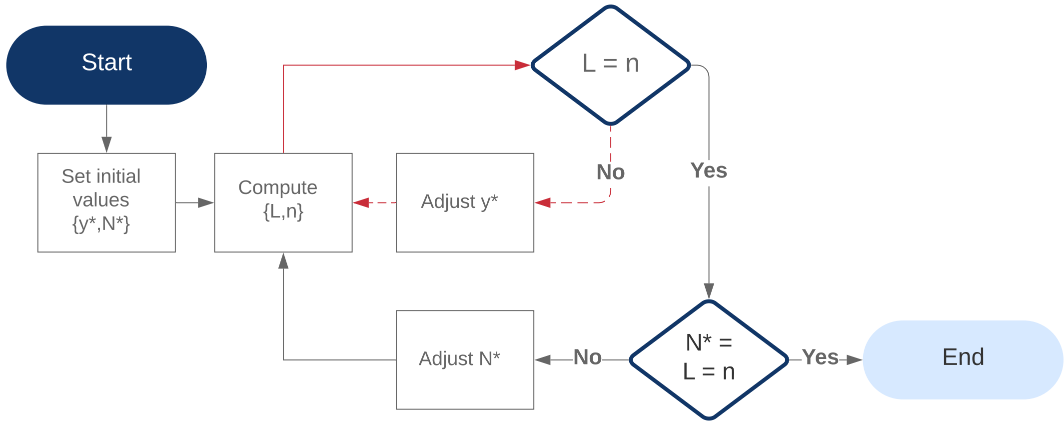
Note: We compute Herfindahl-Hirschman Index (HHI) using the shares of different uses at total floor space for 810,731 buildings in New York City. Source: 2017 NYC PLUTO file from the NYC Dept. of City Planning. Note that data is for virtually all buildings in New York City.

K.2 Solving for the equilibrium

For given values from exogenous parameters $\{\alpha^U, \beta, \omega^U, \theta^U, \tau^U, \bar{a}^U, \tilde{c}^U, \bar{S}^U, \bar{U}\}$, there is a direct mapping from wage y and population N to $\{L(x), n(x), \bar{p}^U(x), r^U(x), \tilde{S}^U(x)\}$ via Eqs. (2), (4), (6), (7), (8), (10), (12), (13). To clear the city's labour market, we set $y^C = y^R = y$ and require that Eq. (14) holds. The city population N enters the production amenity $\tilde{A}^C$ at an agglomeration elasticity β, affecting commercial bid rent in Eq. (4), commercial building height in Eq. (6), land rent in (7), land use in Eq. (8) and, consequentially, labour demand in Eq. (12) and supply in Eq. (13). Because of this simultaneity, we treat the identification of of $\{y, N\}$ as a fixed point problem that we solve using a nested iterative procedure summarized in the programming flow chart in Figure A6.

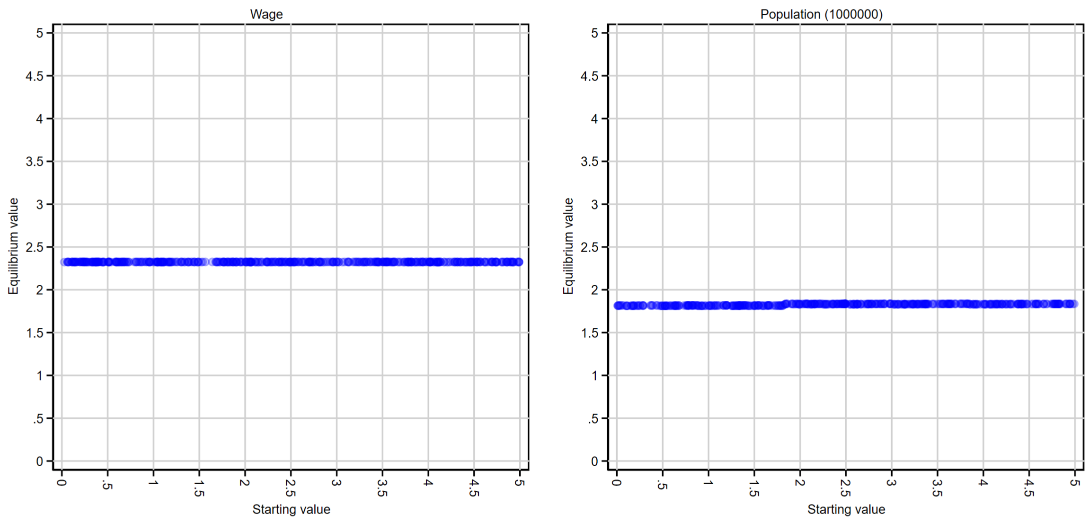
We begin with guessed values for wage and population, which we denote by $\{y^*, N^*\}$. We uses these guesses in Eqs. (2), (4), (6), (7), (8), (10), (12), (13) to compute $\{L(x), n(x)\}$ which we aggregate to city wide labour demand L and labour supply n. Until $L = n$, we adjust y^* using an adjustment factor that is proportionate to the ratio L/n (the red dashed loop). Once the labour market clears, we proceed to comparing the equilibrium employment $L = n$ to our guess N^*. Until $N^* = L = n$, we adjust N^* to a weighted combination of the previous guess and the equilibrium employment obtained from the nested inner loop. Figure A7 shows how this procedure reliably identifies unique equilibrium values that do not depend on starting values. This is no surprise. Labour demand is a negative function of wage and labour supply is an upward sloping function of wage. For plausible values of θ^U the cost of agglomeration exceeds the return to agglomeration. Hence, our numerical solver rapidly converges to an equilibrium.

Figure A6: Programming flow chart



Note: Rectangles indicate processes, diamonds indicate decision rules. * superscirpts indicate guessed values.

Figure A7: Random starting values vs. equilibrium values



Note: We draw random starting values of wage and population from uniform distributions and solve for the equilibrium values 500 times. Each dot represents one combination of starting value and equilibrium value from one of those 500 runs.

K.3 The elasticity of height with respect to land rent

Eq. (5) specifies the developer profit function. Using $\tilde{c}^U = c^U S^U(x)^{\theta^U}$, profit maximization implies the first-order condition:

$$\frac{\partial \pi^U}{S^U} = \bar{p} - c^U(1+\theta^U)(S^U)^{\theta^U} = 0 \tag{15}$$

Solving Eq. (15) for $\bar{p} = c^U(1+\theta^U)(S^U)^{\theta^U}$ and substituting into Eq. (5) delivers the equilibrium relationship between height $S^U = (\theta^U)^{-\frac{1}{1+\theta^U}} r^{\frac{1}{1+\theta^U}}$ and land rent from which we can derive the elasticity of height with respect to land rent as

$$\kappa^U = \frac{1}{1+\theta^U}.$$

Thus, a greater height elasticity of construction cost implies a smaller elasticity of height with respect to land rent and vice versa.

K.4 Related research

Height is typically not modeled explicitly in urban economics models. Yet, urban economics models of the housing supply side typically feature some notion of structural density, broadly defined as housing services per land unit (Epple et al., 2010). Housing developers optimally adjust the use of capital and land to produce housing, leading to higher structural densities. While structural density is technically closer to the floor-area-ratio (FAR) than height, the two measures are mechanically correlated since there are natural bounds for the site occupancy index.

A range of models has linked the supply side to the demand side to rationalize the internal structure of cities. To this end, the monocentric city model has been a workhorse tool in urban economics for at least half a century. As in our model developed in Section 3, spatial competition leads to bid rents that decline with distance from the CBD to offset for transport cost. The canonical Brueckner (1987) version of the model, which draws from Alonso (1964), Mills (1967), and Muth (1969), features a supply side in which profit-maximizing developers respond to changes in bid rents by providing structural densities that decline with the distance from the CBD. Similar to our model, the market-clearing condition can be used to determine either the utility of residents in a city or the population size of a city, depending on whether the open or closed-city model is employed. However, land use segregation is not a feature of this class of models that focus on the housing sector.

More recent models of internal city structure such as Fujita and Ogawa (1982) and Lucas and Rossi-Hansberg (2002) account for the spatial distribution of land uses but exclude the housing and office supply side. Grimaud (1989) shows how to incorporate a housing supply side into a framework akin to Fujita and Ogawa (1982). In the quantitative spatial model of internal city structure developed by Ahlfeldt et al. (2015), land use is endogenously determined with developers producing structural density. Still, since the cost of height is not use-specific, the horizontal land use pattern is independent of the supply side and the vertical dimension of cities. Most models of internal city structure also do not differentiate between within- and between- and building transport cost (Sullivan, 1991) and do not take into within-building spillovers that may arise if skyscrapers promote interactions (Helsley and Strange, 2007).

A notable exception in the theoretical urban economics literature is the model by

Henderson et al. (2021), which not only incorporates height but also distinguishes between a building technology for the formal (tall and durable) and the informal (flat and malleable) sector. Their model predicts that in developing cities, land will be developed informally first, and then formally, with periodical adjustments to changing economic circumstances. Curci (2017) is an example of how to model the housing supply side through convex cost of height in a monocentric city model that is nested in a Rosen-Roback type spatial equilibrium framework. Albouy et al. (2020) provide a theoretical model that links city population to building heights via land prices.

K.5 Potential for future research

One avenue for future theoretical research is to incorporate height-related agglomeration and dispersion forces into models of internal city structure. Our model allows for use-specific costs of and returns to height in a stylized linear city. A natural next step will be to expand quantitative spatial models in the tradition of Ahlfeldt et al. (2015) to incorporate use specific costs of and returns to height. This class of models features a realistic geography and endogenous agglomeration spillovers between nearby locations. Marrying our model of vertical and horizontal spatial structure with the canonical quantitative spatial model would allow for a quantitative evaluation of the role costs of and returns to height play in shaping polycentric city structures.

Going one step further, the model could be extended to account for within-building agglomeration spillovers. The agglomeration force would depend on the height of the building and come in addition to the floor based height premium. While the identification of a building-level agglomeration force is more challenging than the estimation of a within-building height gradient, there is emerging evidence that within-building agglomeration matter (Liu et al., 2020; Curci, 2020).

L Costs of height

This section complements Section 4.1 in the main paper. We present estimate of the change in the height elasticity of construction cost over time, review the related research in greater detail and lay out potential for future research.

L.1 Quantifying the effects of innovation on the cost of height

In Section K.3, we show that there is a mapping from the height elasticity of construction cost θ^U to the elasticity of height with respect to land price κ^U and vice versa. Implcitly, we have assumed an elasticity of substitution between land and capital of $\sigma^U = 1$ and height elasticity of of the share o developable lot space of $\lambda^U = 0$. In a more general supply-side framework, Ahlfeldt and McMillen (2018) derive the more general formulation for the elasticity of height with respect to land rent as $\kappa^U = \frac{\partial \ln S^U}{\partial \ln r^U} = \frac{\sigma^U}{1+\theta^U-\lambda^U}$, which implies that

$$\kappa^U = \frac{\partial \ln S^U}{\partial \ln r^U} = \frac{\sigma^U}{1 + \theta^U - \lambda^U}. \tag{16}$$

We use this formula to recover the height elasticity of construction cost at different point in times from the estimates reported in Figure 7. In Table A5, we then regress the log of the height elasticity of construction cost against a trend time variable, weighting observations by the inverse of the standard errors and controlling for period effects in the case of Chicago. We obtain similar results using the more restrictive parametrization from our Model $\{\sigma^U = 1, \lambda^U = 0\}$) as well as the parameter values $\{\sigma^C = 0.66, \sigma^R = 0.61, \lambda^C = 0.15, \lambda^R = 0.1\}$ from Ahlfeldt and McMillen (2018). Our tentative interpretation of the results is that the cost of height decreased by about 2% per year over the course of the 20[th] century, although we stress that this interpretation hinges on assuming constant values for the elasticity of substitution between land and capital and the height elasticity of extra space.

Table A5: Cost of height over time

	(1)	(2)	(3)	(4)	(5)	(6)
	Log of height elasticity of per-unit cost					
	$\ln(\theta^{(C,R)})$	$\ln(\theta^C)$	$\ln(\theta^R)$	$\ln(\theta^{(C,R)})$	$\ln(\theta^C)$	$\ln(\theta^R)$
Year	-0.022*	-0.016**	-0.015**	-0.023*	-0.018*	-0.017***
	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)
σ	1.00	1.00	1.00	0.66	0.61	0.66
λ	0.00	0.00	0.00	0.15	0.10	0.15
Period effect	Yes	Yes	-	Yes	Yes	-
Observations	11	11	12	11	11	12
R^2	.817	.675	.345	.826	.595	.355

Notes: Unit of observation is decade. Elasticity of per-unit contraction cost with respect to height inferred from the elasticity of height with respect to land price estimates reported in Figure 7 using Eq (16) and parameter values from Ahlfeldt and McMillen (2018). Observations are weighted by the inverse of the standard errors in Figure 7. Period effects control for level shifts in 1920 and 1957 owing to changes in zoning regime in Chicago. + $p < 0.15$, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

L.2 Related research

To avoid hyper-concentration of economic activity into a singular point, urban economics models require a dispersion force. Inelastically provided land represents a natural source of such a dispersion force. In models that incorporate a housing supply side, the amount of usable housing services is not per-se limited. The dispersion force then emerges from marginal costs of housing services that increase in structural density.

It is conventional in this literature to assume a Cobb-Douglas housing production function in which developers produce housing services H rented out at price p using capital and land L as inputs at the factor shares δ and $1 - \delta$. The first-order conditions of profit-maximization along with perfect competition and zero profits deliver an intensive-margin housing supply elasticity of $\frac{d \ln H/L}{d \ln p} = \frac{\delta}{1-\delta}$ (Epple et al., 2010; Ahlfeldt and McMillen, 2020; Baum-Snow and Han, 2019; Combes et al., 2021). Since under the assumptions made, there is a one-to-one mapping from marginal costs c to rents, the implied cost elasticity of structural density is simply the inverse of the housing supply elasticity, i.e. $\frac{d \ln c}{d \ln H/L} = \frac{1-\delta}{\delta}$. While structural density is not the same as height, they are correlated, especially for taller buildings where the site occupancy index varies less. For typical land shares in the range of 10% to one third, the implied cost elasticity of structural density is in line with our estimates of the height elasticity of construction cost reported in Figure 6. There is a debate, however, whether the Cobb-Douglas formulation which implies an elasticity of substitution between land and capital of $\sigma = 1$ is an appropriate approximation for the housing production function. While the literature has not achieved consensus on this question it appears that σ is close to one for smaller structures (Epple et al., 2010; Ahlfeldt and McMillen, 2020; Baum-Snow and Han, 2019; Combes et al., 2021) but smaller than one for tall buildings (Ahlfeldt and McMillen, 2018; Albouy et al., 2020).

There is less economics research that explicitly focuses on the cost of height. Arguably, the first detailed research on the economics of skyscraper height was that from Clark and Kingston (1930). In their work, the authors play the role of a hypothetical developer in Manhattan to investigate the skyscraper height that produces the highest return on investment. They cost out buildings of different heights on the same lot to see how the costs change with height. With this approach, they find that for a large lot in midtown Manhattan, using land values and prices from 1929, the profit-maximising building height was 63 stories. Average total construction costs (total cost divided by gross building area) are minimized at 22 stories, after which they increase at an increasing rate. Based on their costings, we estimate an elasticity of average cost for gross floor space with respect to height of 0.17.

More recent work on the shape of skyscraper cost functions has mostly focused on data from Hong Kong and other Asian cities. This work is summarized in Picken and Ilozor (2015). They discuss the various studies which aim to find the height where average costs are minimized. Interestingly, different studies find different turning points, which range from 30 meters to 100 meters. However, all studies show that after a minimum point, costs rise with height.

Most closely related to our estimates of the cost of height are Ahlfeldt and McMillen (2018) who also exploit the Emporis data set. Employing a more restrictive parametric estimation approach and extending the sample to very tall buildings they estimate somewhat larger height elasticities of construction cost.

Outside economics, there is a literature that provides engineering cost estimates. The rule of thumb is that construction costs tend to increase by 2% per floor (Department of the Environment, 1971), which is in line with more recent estimates (Tan, 1999; Lee et al., 2011).

There have been few works in economics exploring the rate of technological improvements in skyscrapers over time. This may be in part due to the difficulty of getting detailed data to estimate total factor productivity (TFP), for example. Skyscraper developers tend to keep their cost data private. For that matter, they do not tend to itemize costs in a way that readily lend themselves to estimating production functions, which require estimates of the quantities used of labor, natural resources, and capital.

Gort et al. (1999) measure the rate of technological change in building structures by using a vintage capital model, where technological progress is embodied in the form of new capital goods, namely, equipment and structures. Using estimates from a panel data set of 200 office buildings in the United States, they find technological progress increased the TFP of the U.S. construction sector at a rate of 1% per year, and a contribution of 15% of GDP growth over the 1988-1996 period. Chau and Walker (1988) infer TFP growth in the Hong Kong construction industry using various construction cost and price indexes. Their TFP index fell in the 1970s but rose to be about 30% higher by 1984. Other studies confirm that there has been significant TFP growth in building construction (Zhi et al., 2003; Abdel-Wahab and Vogl, 2011; Chau et al., 2005; Wang et al., 2013).

L.3 Potential for future research

The economics literature on the cost of height is still at an early stage. There are few empirical estimates of how construction costs depend on building heights. In the Emporis data used here and in Ahlfeldt and McMillen (2018) is arguably the most comprehensive database on tall buildings used in economics research so far. However, construction costs are missing for about 85% of the buildings, so there is a natural concern about sample selection. More estimates of the cost of height are needed, ideally from more comprehensive samples and for different classes of buildings. It is likely that the cost of height is larger for smaller lot sizes, and there are likely threshold effects in marginal costs at certain heights, but these are difficult to evaluate with the relatively sparse data that have been used so far.

A better understanding of how costs change with respect to building height may inform a broader literature in urban economics concerned with the supply side of housing. This literature is far from reaching a consensus on how substitutable land and capital are in the production of housing. One line of potential research could investigate more systematically how the elasticity of substitution between land and capital changes as places get denser and buildings get taller.

There is a severe lack of evidence on how the cost of height has changed over time. Technological progress in the construction of tall buildings is obvious, and our indirect estimates suggest a sizable reduction in the cost of height over time. To the extent that historical records on construction costs may become accessible, estimating the change in the cost of height over time is a priority area for research into the vertical dimension of cities. We require reliable estimates of the change in construction cost to distinguish between demand-side and supply-side forces that have shaped many skylines over the 20th century.

M Returns to height

This section complements Section 4.2 in the main paper. We derive an estimation equation for the height elasticity of rent, discuss the height elasticity of average rent in samples with buildings with finite height, review the related research in greater detail and lay out potential for future research.

M.1 Estimating the height elasticity of rent

In this section, we derive an empirical specification suitable for the estimation of the height elasticity of unit rent from our model and discuss how our estimates relate to the height elasticity of average building rent in a sample of buildings with a finite number of floors.

M.1.1 Deriving an estimation equation

In Section 3, we have derive the residential bid rent by horizontal location x and vertical location s as

$$p^R(x,s) = A^R(x,s)^{\frac{1}{1-\alpha^R}} y^{R\frac{1}{1-\alpha^R}} \tag{17}$$

and the commercial bid rent as

$$p^C(x,s) = A^C(x,s)^{\frac{1}{1-\alpha^C}} y^{C\frac{\alpha^C}{\alpha^C-1}} \tag{18}$$

Using $A^R(x,s) = \tilde{A}^R(x)s^{\tilde{\omega}^R}$, $a^R(x) = \tilde{A}^R(x)^{\frac{1}{1-\alpha^R}} y^{R\frac{1}{1-\alpha^R}}$, $\omega = \frac{\tilde{\omega}}{(1-\alpha^R)}$, $A^C(x,s) = \tilde{A}^C(x)s^{\tilde{\omega}^C}$, $a^C(x) = \tilde{A}^C(x)^{\frac{1}{1-\alpha^C}} y^{C\frac{\alpha^C}{\alpha^C-1}}$ and $\omega^C = \frac{\tilde{\omega}^C}{1-\alpha^C}$ in Eqs. (19) and (18), we can express the use-specific bid rent as:

$$p^U(x,s) = \bar{a}^U(x)s^{\omega^U} \tag{19}$$

Log-linearization motivates a straightforward estimation equation

$$\ln p^U_{i,b,t} = \omega^U \ln s_i + a^U_b + b^U_t + \epsilon^U_{i,b,t}, \tag{20}$$

where $p^U_{i,b,t}$ is the observed market rent of a unit i in building b at time t, a^U_b is a building fixed effect that summarizes the effects of all location-specific factors, b^U_t is a time fixed effect that captures the time trend, and $\epsilon^U_{i,b,t}$ is an error term that captures idiosyncratic factors. Note that based on an estimate of the height elasticity of rent ω^U, it is straightforward to recover the height elasticity of amenity as

$$\tilde{\omega}^U = (1-\alpha^U)\omega^U.$$

M.1.2 The height elasticity of average rent with finite building height

In our model, the relationship between building height S^U and the average rent within a building $\bar{p}(x)$ is moderated by the same height elasticity of rent $\omega^U = (1-\alpha^U)\tilde{\omega}^U$ that

Table A6: Height elasticity of average building rent

Use (U)	Commercial			Residential		
Floors (S^{MAX})	10	40	100	10	40	100
$\hat{\omega}^U$ (height elasticity of unit rent)	0.033	0.033	0.033	0.073	0.073	0.073
ω^U (height elasticity of building rent)	0.022	0.026	0.029	0.047	0.056	0.062
$\omega^U/\hat{\omega}^U$	0.667	0.788	0.879	0.644	0.767	0.842
$\tilde{\omega}^U$	0.005	0.005	0.005	0.024	0.024	0.024
$1-\alpha^U$	0.150	0.150	0.150	0.330	0.330	0.330

Notes: Estimates of $\hat{\omega}^U$ from Figure 8. The height elasticity of unit rent moderates how the rent changes across floors within a building. The height elasticity of the average building rent moderates how the average building rent changes in the height of the building. For infinitely tall buildings, both elasticities converge to the same value.

also relates the within-building unit rent s to the unit rent $p(x,s)$. This is true because we implicitly assume that a move by one floor within a building is a marginal change. In reality, changes in height are typically non-marginal due to the integer floor constraint so that the two elasticities are only asymptotically identical for very tall buildings.

Therefore, the height elasticity of unit rent and the average height elasticity of average building rent are distinct objects, empirically. To convert an empirical estimate of the height elasticity of unit rent from Eq. (20) into an estimate of the height elasticity of average building rent, we proceed as follows. We generate a series of artificial buildings with $S^U \in (1, 2, ..., S^{MAX})$ and compute the aggregate rental revenue per land unit as

$$R^U_{S^U} = \sum_{s=1}^{S^U} p^U s^{\hat{\omega}^U},$$

where $\hat{\omega}^U$ is our estimate of the height elasticity of unit rent from Eq. (20) reported in in Figure 8. To obtain an estimate of ω^U that corresponds to Eqs. (2) and (4) in spirit, we regress the log of $R^U_{S^U}$ against the log of S^U.

We present the results of this exercise in Table A6. In keeping with intuition, the height elasticity of average building rent is smaller than the height elasticity of unit rent, but the former converges to the latter as we expand the sample of synthetic buildings to include taller buildings. So, in a vertical city, the latter represents a decent approximation for the former. For flat cities, transforming $\hat{\omega}^U$ into a feasible approximation of ω^U is appropriate. The ratios $\omega^U/\hat{\omega}^U$ reported in Table A6 provide some guidance.

M.2 Related research

Recent empirical work substantiates our empirical finding that there is a rent premium at higher floors. Unlike in the above estimates, however, many existing studies do not control for building fixed effects. This is a limitation since estimates of vertical rent gradients may be confounded by attributes of horizontal space that may correlate with average building heights, such as distance from the CBD.

Koster et al. (2013) study rents in Dutch office buildings. They find that firms are willing to pay a 4% premium to be in a building that is 10 meters taller. Their findings suggest that the premium comes from a mix of agglomeration benefits, views, and the status associated with working in a structure that stands out in the skyline (i.e., a "landmark" effect). Shilton and Zaccaria (1994) and Colwell et al. (1998) similarly find higher office rents in taller buildings. An exception in this literature is Eichholtz et al. (2010), who find mixed results. Compared to previous work, Liu et al. (2018) improve on the identification in that they control for building fixed effects when estimating commercial vertical rent gradients across multiple U.S. cities. They estimate significantly larger values of the floor elasticity of rent β^C of 0.086 within the CompStat data set and even 0.189 within the Offering Memos data set. Using the same procedure as above, we can translate these estimates into estimates of the height elasticity of average rent ω^C of 0.07 and 0.15 for buildings up to $S^{MAX} = 40$ floors.

As for residential height premiums, Wong et al. (2011) find a vertical price gradient in Hong Kong apartments, while Chau et al. (2007) find a price premium in Hong Kong for those units that have a sea view. Danton and Himbert (2018) estimate that within buildings in Switzerland, residential rents increase by 1.5% per floor. Their sample mostly consists of smaller structures. For the average floor of two within their sample, the implied floor elasticity of rent β^R is 0.03, somewhat less than what we find of taller residential structures in New York City and Chicago.

Some of the more recent studies on vertical rent gradients have attempted to identify the underlying mechanisms. In perhaps the first paper to include a measure that controls for views, Nase et al. (2019) find that for the Amsterdam office market, 27% of the height premium is related to the view, while 70% is due to firm-level signaling and other firm-specific factors. Liu et al. (2018) find evidence that firms in the U.S. pay premiums not only for better amenities (views and sunlight) but also to signal their productivity. They also document vertical sorting, with businesses that generate higher revenues per worker, such as law firms, being located on higher floors.

In contrast, businesses that value accessibility, such as retail, locate on lower floors, even paying a significant ground floor premium. Related to the "signaling effect," Dorfman et al. (2017) perform a novel behavioral study in which they explore the perceptions of status and building height. In particular, they investigate how people view the concept of power in regards to floor height. From their survey analysis, they find that people perceive those residing on higher floors as being more powerful. Ben-Shahar et al. (2007) apply a cooperative game theory model to allocate the land and construction costs among the stories of the building. They show how the desire for status can generate a height premium in the cost allocation game.

Liu et al. (2020) show that the vertical employment density gradient follows the vertical rent gradient, just like the horizontal density gradient follows the horizontal rent gradient. Hence, there is a u-shaped relationship between worker density and height, consistent with firms optimally adjusting factor inputs to factor prices.

M.3 Potential for future research

Despite recent progress, there are still relatively few studies that explore vertical rent gradients controlling for location via building effects. In particular, the evidence is thin for commercial buildings outside the U.S. and tall residential buildings more generally.

There is no substantial evidence on how returns to height have changed in the long run. In determining optimal building heights, returns to height are isomorphic to the costs of height. Hence, in terms of rationalizing the evolution of the urban height gradient over time, understanding changes in returns to height is as important as understanding changes in the cost of height.

The origins of the residential vertical rent gradient have remained understudied. Disentangling the effect of a view amenity from other height effects such as prestige is a fairly obvious research question. Similarly, little is known about heterogeneity in the valuation of the height amenity. Estimates of the income elasticity of the height amenity would be informative with respect to vertical and horizontal sorting. If richer households were willing to pay a greater height premium, tall buildings could contribute to spatial income segregation through a preference channel, in addition to an affordability channel.

Despite the notion that certain agglomeration effects such as knowledge spillovers are highly localized (Rosenthal and Strange, 2001), productivity spillovers within buildings have remained under-researched. Decades of research into horizontal agglomeration effects naturally suggest that priority areas for research into vertical spillovers should include causal estimates of the vertical within-building spillovers after controlling for selection; the attenuation of spillovers over vertical distances; the relative importance of information and input sharing, matching, and learning within buildings; and the co-location benefits across industries (see Combes and Gobillon (2015) for a recent review of the empirics of agglomeration).

N Vertical and horizontal city structure

This section complements Section 5 in the main paper. We provide additional detail on the parametrization of our model, our empirical approach to measure the discontinuity in the height gradient at the land use boundary and how we account for the fuzziness of the height gradient. Then, we review the related literature and lay out potential for future research.

N.1 Parametrization

N.1.1 Spatial decay in production amenity

Eq. (4) states that the commercial bid adjusts to offset for changes in attractiveness of location captured by $\bar{a}^C(x)$. This is a standard prediction of competitive spatial equilibrium models. A novel feature of Eq. (4) is that the horizontal bid rent $\bar{p}^c(x)$ also depends on building height $S^C(x)$ due to the height amenity effect. Within our model, the optimal building height $S^{*C}(x)$ is an endogenous outcome that itself depends on the horizontal bid rent $\bar{p}^c(x)$ and, indirectly, the production amenity of location x. Using $S^{*C}(x) = \left(\frac{a^C(x)}{c^C(1+\theta^C)}\right)^{\frac{1}{\theta^C-\omega^C}}$ and $a^C(x) = \tilde{A}^C(x)^{\frac{1}{1-\alpha^C}} y^{C\frac{\alpha^C}{\alpha^C-1}}$ in Eq. (4) and taking logs, we obtain

$$\ln \bar{p}^C(x) = \xi + \frac{\tau^C \theta^C}{(1-\alpha^C)(\theta^C - \omega^C)} D(x),$$

where ξ collects an array of log constants. From this equation, we derive the reduced-form empirical specification:

$$\ln \bar{p}^C_{g,m} = b \ln D_{g,m} + \xi_m + \epsilon^C_{g,m}, \tag{21}$$

where $\bar{p}^C_{g,m}$ is the average office rent within a 250×250 meter grid cell in city m, ξ_m is a city fixed effect, $D_{g,m}$ is distance from the city center approximated by a global prime location identified by Ahlfeldt et al. (2020), and $\epsilon^C_{g,m}$ is an error term capturing unobserved location and building characteristics. From an estimate of the reduced-form parameter b, we can recover the structural parameter

$$\tau^C = b\frac{(1-\alpha^C)(\theta^C - \omega^C)}{\theta^C}.$$

We report the reduced-form estimates of b and the implied values of the structural parameter τ^C in Table A7. We find that, on average, commercial rents decrease by slightly less than 8% per kilometer distance from a prime location. The rate of decay is almost twice as large within our sample of 21 European cities than within our sample of 24 U.S. cities.

It is worth noting that Rosenthal et al. (2021) find a pre-Covid effect from distance to CBD of 0.013 (converted from miles into kilometers) for a much larger sample of 109 U.S. cities. At 0.04 (again converted into per-kilometer terms), their estimates are closer to ours for seven transit-oriented cities. Hence, some of the difference between the estimates for U.S. cities is likely due to the selection of cities in our sample. However, even conditional on the same sample of cities, the results would likely differ since the SNL-S&P data we use draws mostly from buildings held by Real Estate Investment Trusts whose assets are highly selective (almost exclusively grade-A buildings). Moreover, Ahlfeldt et al. (2020) and Rosenthal et al. (2021) use different algorithms to identify prime locations and CBDs, so that the nuclei of the rent gradients are not necessarily the same.

N.1.2 Land rent gradient in model

The analytical solution to the elasticity of land rent with respect to distance from the core is complex and varies by distance and land use. For a simple approximation of the average elasticity, we regress the log of land rent plotted in Figure 10 against the log of the distance from the core and report the results in Table A8. The -0.56 estimate corresponds to the estimate Ahlfeldt and McMillen (2018) report for Chicago in the 1960s. They report lower values for 1960s, 70s and 80s and higher values for the other decades of the 20$^{\text{th}}$ century.

Table A7: Floor space rent gradients

	(1) Ln office rent	(2) Ln office rent	(3) Ln office rent
Distance from prime location (km)	-0.077***	-0.072***	-0.134***
	(0.01)	(0.02)	(0.02)
City fixed effects	55	24	21
Region	World	North America	Europe
τ^C	-.011	-.01	-.019
Observations	357	138	154
R^2	.625	.527	.549

Notes: Unit of observation is 250x250-meter grid cells. Rents are in per unit terms and obtained at the level of individual buildings from SNL-S&P Global before aggregating them to grid cells. Prime locations are from (Ahlfeldt et al., 2020). North American cities include: Atlanta, GA, Austin, TX, Boston, MA, Chicago, IL, Cincinnati, OH, Dallas, TX, Denver, CO, Fort Worth, TX, Houston, TX, Indianapolis, IN, Los Angeles, CA, Minneapolis, MN, Montreal, Canada, New York, NY, Orlando, FL, Philadelphia, PA, Phoenix, AZ, Pittsburgh, PA, Saint Louis, MO, San Diego, CA, San Francisco, CA, Seattle, WA, Tampa, FL, Washington, DC. European cities include: Amsterdam, Netherlands, Antwerp, Belgium, Athens, Greece, Berlin, Germany, Brussels, Belgium, Budapest, Hungary, Düsseldorf, Germany, Frankfurt, Germany, Gothenburg, Sweden, Hamburg, Germany, Helsinki, Finland, Linköping, Sweden, London, United Kingdom, Lyon, France, Malmö, Sweden, Munich, Germany, Paris, France, Rotterdam, Netherlands, Stockholm, Sweden, Vienna, Austria, Zurich, Switzerland. Other cities include: Cape Town, South Africa, Durban, South Africa, Fukuoka, Japan, Hong Kong, China, Johannesburg, South Africa, Nagoya, Japan, Osaka, Japan, Rio de Janeiro, Brazil, Sao Paulo, Brazil, Tokyo, Japan. $^{+}$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A8: Land rent gradient in model

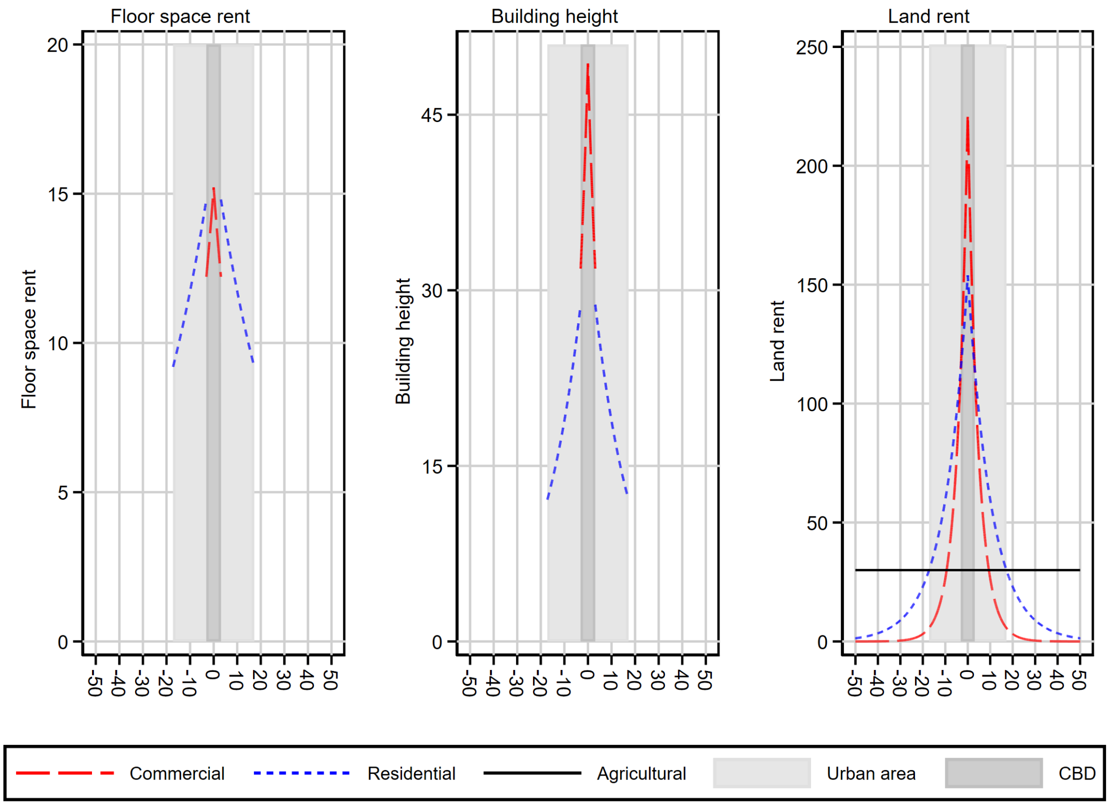
	(1) Ln land rent	(2) Ln land rent	(3) Ln land rent
Ln distance from city core	-0.563***	-0.280***	-0.774***
	(0.01)	(0.01)	(0.00)
Use	All	Commercial	Residential
Observations	6908	1012	5896
R^2	.908	.77	.947

Notes: Unit of observation is 0.01 distance grid cells in the model. Standard errors in parentheses. sym+ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

N.1.3 Alternative parametrizations

In this Section we replicate Figure 10 under different parametrizations to strengthen the intuition for how the amenity decay τ^U and the height elasticities $\{\theta^U, \omega^U\}$ jointly shape the horizontal and vertical spatial structure. Figure A8 shows how increasing the residential amenity decay compresses the residential zone. Greater competition of residential space pushes the maximum residential rent to nearly the level of the maximum commercial rent. Yet, commercial buildings remain significantly taller. Figure A9 illustrates how the discontinuities in the floor space rent and height gradients at the land use boundary disappear once the commercial and residential height elasticities $\{\omega^U, \theta^U\}$ are set to uniform values.

Figure A8: Urban gradients: Uniform amenity decay

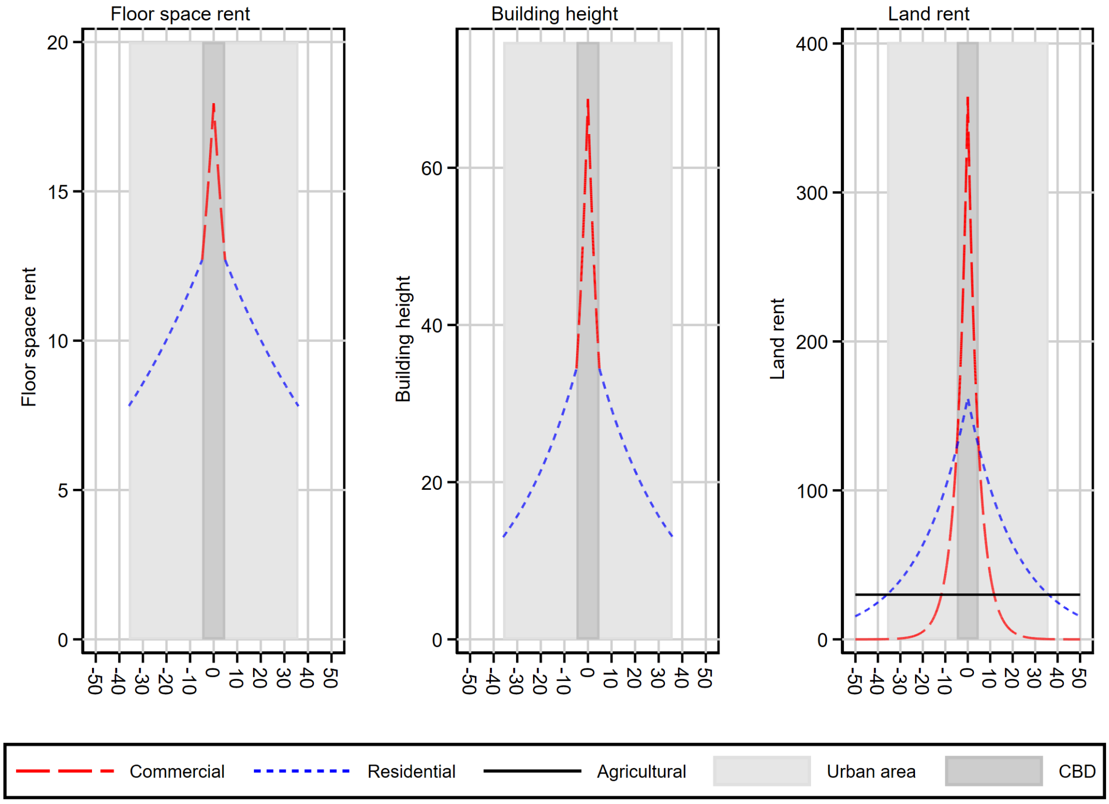


Note: Figure illustrates the solution to the model laid out in Section 3 using the parameter values reported in Table 1) with the following expiation: The residential amenity decay τ^R is set to the value of the production amenity decay $\tau^C = 0.01$.

N.2 Height discontinuity at CBD boundary

To sensibly approximate the boundary of the commercial and residential zones, we focus on North American cities with at least 100 residential and 100 residential buildings in the Emporis data. To approximate the land use boundary, we compute the residential and

Figure A9: Urban gradients: Uniform height elasticities



Note: Figure illustrates the solution to the model laid out in Section 3 using the parameter values reported in Table 1) with the following expiation: The residential height elasticity of rent ω^R and the residential height elasticity of construction cost θ^R are set to the values of the commercial elasticities $\omega^C = 0.03, \theta^C = 0.5$.

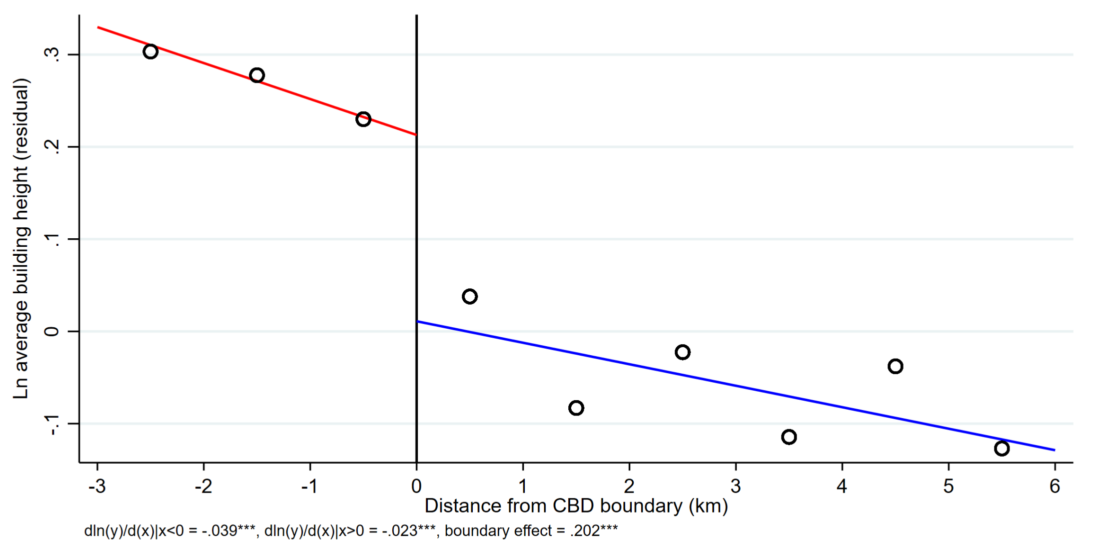
commercial building density by one-kilometer distance-from-from-the commercial-center rings for each city. The commercial centers are the largest prime locations identified by Ahlfeldt et al. (2020). Starting from the commercial center, the first ring where the residential density exceeds the commercial density, marks the beginning of the residential zone. Having identified the land use boundary, we estimate the discontinuity in the height gradient using the following boundary discontinuity design:

$$\ln H_{i,m} = d_1 \mathbb{1}(D_{i,m} < \bar{D}_m) + d_2(D_{i,m} - \bar{D}_m) + d_3(D_{i,m} - \bar{D}_m) \times \mathbb{1}(D_{i,m} < \bar{D}_m) + \xi_m + \epsilon_{i,m}$$

where $D_{i,m}$ is the distance of a building i from the commercial centre of city m, $\bar{D}_m$ defines the land use boundary in terms of the city-specific distance from the commercial centre, ξ_m is a city fixed effect, and $\epsilon_{i,m}$ is an error term. The parameter of interest is d_1, which captures the average difference in building heights just inside and outside the land use boundary.

We illustrate the results graphically in Figure A10. As one steps inside the CBD, building heights increase on average by some significant 0.2 log points, consistent with the predictions of our model under the the parameter values summarized in Table 1. Also consistent with the model predictions, the height gradient is steeper within the CBD than outside.

Figure A10: Height discontinuity at CBD boundary



Note: Negative values of the running variable (distance from the CBD boundary) indicate locations inside the CBD. Positive values indicate locations outside. City-specific CBD boundaries are determined as circles with radii defined such that the difference between the density of commercial buildings and the density of residential buildings turns from positive to negative as one crosses the boundary. Ln average building height is residualized to take out city fixed effects. The boundary effect is estimated in a regression of the residualized log of height against an indicator for the location within the CBD, the running variable, and the interction of both. Unit of observation is city × 1-km distance rings. *** indicates significance at the 1-percent level. Data from Emporis. 15 North American cities included with at least 100 tall residential and 100 tall commercial buildings recorded in the data base Atlanta, Chicago, Denver, Houston, Los Angeles, Miami, Minneapolis, Montreal, New York, Ottawa, San Francisco, Seattle, Toronto, Vancouver, Washington). City centers are prime locations from Ahlfeldt et al. (2020).

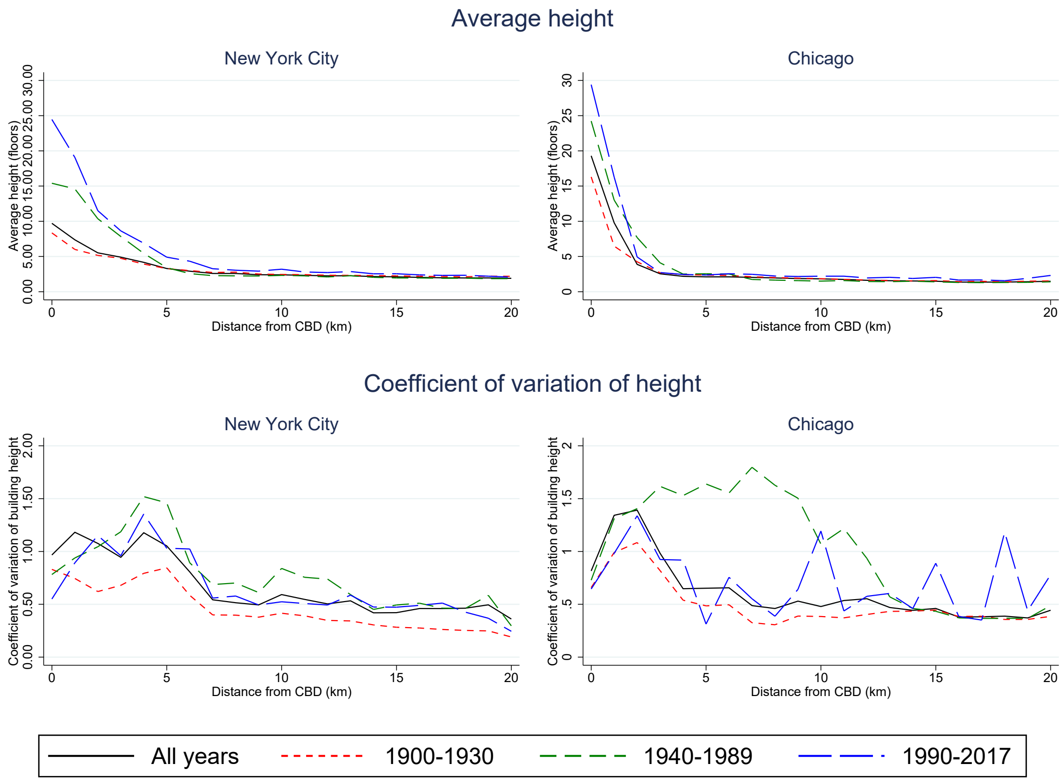
N.3 Within- and between cohort variation in building heights

Figure A11, illustrates average building heights and variation in building heights by one-kilometer bins defined in terms of distance from the CBD for the most vertical U.S. cities.

N.4 Related research

The literature on the evolution of height gradients is still nascent and largely confined to particular cities. For New York City, a few studies have looked at the evolution of building heights and densities across space and time, and how they may be influenced by geology, agglomeration benefits, and other factors. (Barr and Tassier, 2016; Barr et al., 2011; Barr, 2012). Barr and Cohen (2014) study the evolution of the floor area ratio (FAR) gradient for commercial buildings in New York City across both time and space. They find that the FAR gradient for the city as a whole flattened over the first half of the 20^{th} century, then remained relatively steady between the late-1940s and mid-1980s, and then flattened

Figure A11: Average height and coefficient of variation by construction date cohort



Note: CBD in New York City is the Empire State Building. CBD in Chicago is W. Jackson Blvd. and S. Dearborn Streets. Data from the NYC PLUTO File and the Chicago Building Footprints Shapefile. Buildings are grouped into cohorts defined by the construction date.

to a new "plateau" over the last quarter-century.

For Chicago, Ahlfeldt and McMillen (2018) evaluate the evolution of height and land price gradients over time, providing estimates similar to the ones reported here. Henderson et al. (2021) provide a unique analysis of the urban height profile in the context of a developing city. They document that in Nairobi, the built volume in the core city increased by more than 50% over 12 years. They also show that the height gradient is flatter in the informal than in the formal housing sector in Nairobi. This finding is consistent with the lack of capital access among those who build in the informal sectors (Bertaud, 2018). More loosely related, there is an older literature that has estimated the rate at which population density decreases in distance from the CBD summarized in McDonald (1989). Thus, there is an intimate relationship between rent and density gradients, which is consistent with the standard monocentric model (Brueckner, 1987). But as discussed in Bertaud (2018), the negative density gradients can reverse in non-market economies, such as pre-1978 China or the former Soviet Republics, where planning often places high-rise social housing projects far from the city center.

In general, most of the empirical literature focuses on smooth height gradients. We have shown that micro-geographic variation in the amenity value of locations can rationalize fuzzy height gradients. In addition, there are several explanations that have received attention in the economics literature to varying degrees.

Holdouts. Skyscrapers generally require large lots to make the economics work. In dense older cities like New York, lots on a block can be owned by several owners. A developer of a tall building then needs to assemble lots. During this process, owners of one or two strategically-located lots may refuse to sell in an effort to extract monopoly rents (Brooks and Lutz, 2016; Cunningham, 2013; Lindenthal et al., 2017; Strange, 1995). This is the holdout problem. To avoid this problem, developers typically buy lots in secret. Still, if a block has many small lots with different owners, it may take years or decades even to assemble a sufficiently large parcel for the development of a skyscraper.

Durability. Tall buildings by their very nature are frequently expected to last decades if not centuries. Beyond marginal changes, it is not economically feasible to adjust the height of existing tall structures as this would require additional elevator shafts and reinforcements of structural components and foundations. Brueckner (2000) provides a summary of the standard land-use models that incorporate the durable nature of structures. Some models assume that structures are infinitely-lived, and new ones get added on the urban core as the city expands. Other models assume redevelopment based on depreciation and changing prices. Brueckner (2000) and Henderson et al. (2021) nicely illustrate how adding a dynamic element to the land use model can produce "jagged" or fractal spatial structure in regard to its structural density. Glaeser and Gyourko (2005) model how building stock durability affects the nature of cities that decline relative to those that grow.

Option Value An owner of a central-city vacant or under-utilized lot must decide on the timing of construction (Titman, 1985; Capozza and Helsley, 1990; Williams, 1991). Because of the inherent uncertainty and the long lag between ground breaking and opening, developers cannot always correctly predict the revenue they will obtain upon opening. Consistent with options theory, Barr (2010) shows empirically that price uncertainty delays skyscraper construction in Manhattan. Due to strategic interaction, option values that build up over time are capitalized periodically, leading to waves of local development under different fundamentals with correspondingly differing economic heights (Grenadier, 1995, 1996; Schwartz and Torous, 2007).

Institutional factors. There are several institutional explanations for micro-geographic variation in building heights. Land use regulations and historic preservation are obvious candidates. New York City zoning rules, for example, limit the amount of floor area on each block. Owners of smaller buildings with "extra" floor area can sell it to owners of neighboring properties. They then forego any future densification of their properties, and the result is a persistent difference in the height of neighboring buildings. Property rights also matter. Public ownership may deter teardowns since evicting residents can be politically costly. As an example, municipal governments in China typically sign 70-year land leases with residential developers. When these leases run out in the coming decades, governments may face pressure to renew them to avoid redevelopment and mass displacement. Large cities in the U.S. that own public housing projects face similar pressures. The protection of sitting tenants, such as under the U.S. rent control and stabilization laws, implies that landlords cannot redevelop their properties if some tenants exercise their right of lease renewals. If ownership is private but fragmented, such as in a coop, condominium, or homeowner association, selected owners can veto the redevelopment of a property, even if there is a clear economic case.

N.5 Potential for future research

There is further need for additional studies that analyse the evolution of height gradients overtime beyond a case study context. The analysis of within-skyscraper productivity spillovers and transport costs also is a priority area for empirical research into the vertical dimension of cities. In the longer run, evidence may motivate theoretical research to incorporate skyscraper-related agglomeration and congestion forces into models of the internal structure of cities.

While natural amenities, endogenous agglomeration, and transport networks have been explored as sources of persistence in the internal structure of cities (Lee and Lin, 2017; Brooks and Lutz, 2019; Ahlfeldt et al., 2020), the durability of building stock has been overlooked. Skyscrapers typically occupy the most productive urban areas and potentially represent an additional source of path-dependency. At the same time, vintage effects may encourage shifts in the spatial structure of cities as building capital depreciates (Brueckner and Rosenthal, 2009). Hence, aging tall building stock could theoretically promote the emergence of edge cities (Henderson and Mitra, 1996). Since skyscrapers are extremely durable, they may play a significant role in moderating how urban economies shift between multiple steady states. In this context, it may be worth revisiting the current workhorse models with a view to incorporating durable building stock.

Except for Barr (2010), there is no work that empirically explores the timing of skyscraper construction, or how tall building durability might affect future construction. In most larger, older cities, like New York and London, the vast majority of structures were built before World War II. This suggests a tremendous lock-in force at work once the decision to build is made. Skyscrapers rarely get torn down, and the implications for spatial structure, agglomeration effects, and the quality of urban life are worth exploring. Closely related, there is also scope for research linking the increasing option value of vacant land (Barr et al., 2018) to the timing of development, the malleability of urban spatial structure, and the cost of housing and office space.

O Skyscrapers as causes and effects of agglomeration

This section complements Section 6 in the main paper. We derive model-based city size elasticities, review the related research in greater detail and lay out potential for future research.

O.1 Model-based city size elasticities

In Figures 13, 14 and 15, we report the solutions to our model obtained under varying parameter values of $\{\beta, \tau^U, \theta^U\}$. To approximate an average model-based city side elasticity of an outcome, we regress the log of the model-based equilibrium outcome against the log of the equilibrium population obtained for different parameter values. We report the results for different outcomes by the source of variation in Tables A9, A10, and A11.

Table A9: City-size elasticities from variation in returns to agglomeration

	(1) Ln dist. to zone edge	(2) Ln dist. to zone edge	(3) Ln max. rent	(4) Ln max. rent	(5) Ln tallest building height	(6) Ln tallest building height
ln population	0.179***	0.532***	0.331***	0.287***	0.662***	0.521***
	(0.01)	(0.01)	(0.00)	(0.01)	(0.00)	(0.01)
Use	Commercial	Residential	Commercial	Residential	Commercial	Residential
Observations	11	11	11	11	11	11
R^2	.96	.996	1	.996	1	.996

Notes: We correlate the outcomes of model-based equilibrium outcomes solved for varying values of the agglomeration elasticity β. Unit of observation is simulation runs. $^+$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A10: City-size elasticities from variation in amenity decay

	(1) Ln dist. to zone edge	(2) Ln dist. to zone edge	(3) Ln max. rent	(4) Ln max. rent	(5) Ln tallest building height	(6) Ln tallest building height
ln population	0.693***	0.901***	0.154***	0.045***	0.308***	0.082***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)
Use	Commercial	Residential	Commercial	Residential	Commercial	Residential
Observations	11	11	11	11	11	11
R^2	1	1	.996	.992	.996	.992

Notes: We correlate the outcomes of model-based equilibrium outcomes solved for varying values of the amenity decay $\{\tau^C, \tau^R\}$. We keep the relative value constant at $\theta^C - \tau^R = 0.005$ in all simulation runs. Unit of observation is simulation runs. $^+$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

O.2 Related research on skyscrapers as consequence of agglomeration

That population and rents adjust to offset for higher productivity and wages and restore the equilibrium is a standard prediction of spatial equilibrium models in the tradition of

Table A11: City-size elasticities from variation in cost of height

	(1) Ln dist. to zone edge	(2) Ln dist. to zone edge	(3) Ln max. rent	(4) Ln max. rent	(5) Ln tallest building height	(6) Ln tallest building height
ln population	0.046***	0.372***	0.103***	0.116***	0.940***	0.769***
	(0.01)	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)
Use	Commercial	Residential	Commercial	Residential	Commercial	Residential
Observations	21	21	21	21	21	21
R^2	.502	.982	.996	.991	.998	.994

Notes: We correlate the outcomes of model-based equilibrium outcomes solved for varying values of the height elasticity of construction cost $\{\theta^C, \theta^R$. We keep the relative value constant at $\theta^C - \theta^R = 0.05$ in all simulaiton runs. Unit of observation is simulation runs. $^+$ $p < 0.15$, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Roback (1982). The open-city version of the canonical monocentric city model predicts an increase in structural density and rent level as the population increases (Brueckner, 1987; Duranton and Puga, 2015) and so do more recent quantitative spatial models Ahlfeldt et al. (2015). Empirically, the relationship between a population and housing cost has been explored by Combes et al. (2019). A review of the small related literature is provided by Ahlfeldt and Pietrostefani (2019).

Historically, skyscraper development has likely been fueled by an increase in returns to agglomeration as cities became much more interactive and, hence, productive over the 20$^{\text{th}}$ century. The effects of increasing external returns on the horizontal spatial structure have been explored by Ahlfeldt and Wendland (2013) for Berlin and by Ahlfeldt et al. (2020) within a large sample of global cities. Similarly, there is convincing evidence that transport improvements lead to decentralization of economic activity in horizontal space (Baum-Snow, 2007; Baum-Snow et al., 2017; Gonzalez-Navarro and Turner, 2018; Heblich et al., 2020). Much less is known about the effects on the vertical dimension of cities.

The literature that directly addresses building heights within spatial equilibrium frameworks is relatively recent. Curci (2017) models horizontal and vertical spatial structure in an open-city spatial equilibrium framework. Albouy et al. (2020) also present a quantitative framework for the analysis of the relationship between building heights and other important outcomes such as population and land values. They estimate a population elasticity of the height of top-10 buildings of 0.38 across U.S. cities. They argue, that if population only impacts on building heights via the land value, population represents a valid instrument for land value when estimating the elasticity of building height with respect to land value.

Methodologically, the problem of establishing a causal effect of urbanization on vertical growth is related to the estimation of the supply elasticity of housing, which moderates how the supply of housing responds to increasing demand. Saiz (2010) uses a Bartik (1991) instrument as a demand-side instrument for the estimation of the supply elasticity. Related studies include Mayer and Somerville (2000), Green et al. (2005), Hilber and Mayer (2009) and Baum-Snow and Han (2019). Epple et al. (2010), Ahlfeldt and McMillen (2020) and Combes et al. (2021) take a more structural approach and treat the price and quantity of housing services as latent variables which are structurally related via the production function for housing.

O.3 Related research on skyscrapers as cause of agglomeration

Intuitively, skyscrapers are so tall and heavy that they need to be stabilized in some way so that they do not lean or deferentially settle (or fall over). This intuition underpins the use of geological conditions as relevant instruments for density in the identification of agglomeration spillovers (Rosenthal and Strange, 2008; Combes et al., 2011). Curci (2020) uses a shift-share approach where the interaction between distance to bedrock and the steel price serve provide time-varying variation to identify agglomeration effect. The idea to use sub-soil conditions for the identification of agglomeration effects is compelling since direct effects on the demand side of urban land markets are not immediately apparent. Yet, the idea is not uncontroversial.

Deeply embedded in New York's historiography is that skyscrapers are "missing" between Downtown and Midtown because the bedrock in that area (which includes Greenwich Village and the Lower East Side) is far below the surface. From a detailed inspection of building heights and bedrock depths, however, Barr et al. (2011) conclude that subsoil supply-related conditions play a much less important role for skyscraper formation than demand for space in the two economic centers. Where engineers encountered difficulties regarding the geological conditions, they were able to devise innovative methods to make skyscrapers cost effective. That said, there is some evidence that, conditional on the decision to develop a skyscraper within Downtown, developers may have "moved" their skyscrapers to locations with more favorable geological conditions. So, bedrock appears to be a relevant determinant of building heights in New York at a micro-geographic level primarily.

Closely related, Barr and Tassier (2016) show that the rise of Midtown as a separate skyscraper district was due to the agglomerative forces related to shopping and commercial activity north of 14$^{\text{th}}$ Street. This commercial district emerged after the U.S. Civil War, following the northward expansion of Manhattan's residential districts. Having become the natural focal point on a long and narrow island, Midtown was the second location after Downtown where demand drove rents above the threshold that made skyscrapers economically viable. Taken together, Barr et al. (2011) and Barr and Tassier (2016) demonstrate that Manhattan's skyline shape was driven by the demand side and not the supply side.

The evidence does not substantiate that bedrock depths are a major determinant of building heights in New York. To rationalize this result, it is worth considering that Chicago, another early adopter of the skyscraper technology, was built on "swampy soil" (Bentley and Masengarb, 2015). Thus, bedrock is not a binding requirement for the construction of skyscrapers. Although skyscrapers are anchored directly to the bedrock where it is easily accessible, there are relatively cost-effective alternatives. Typically, if there is no bedrock near the surface, foundations are created by boring long piles into the subsoil and then placing a concrete mat on top of them, with the structure built on top of the mat. It is not clear that anchoring a skyscraper to bedrock is necessarily always the cheaper alternative. As an example, accessing bedrock in downtown Manhattan is quite difficult because on top of the rock floor is viscous, wet quicksand with boulders scattered throughout. Engineers had to devise expensive caisson technology in order to build foundations in lower Manhattan (Barr, 2016). And even if establishing the foundations for a tall building on bedrock is cheaper, the cost of stabilizing a building may not vary greatly in height. In fact, in 1929 Manhattan foundation costs were found to be essentially independent of height (Clark and Kingston, 1930).

It seems possible that conventional wisdom overstates the role of bedrock in facilitating skyscraper development since engineering offers suitable alternatives. Yet, this concern primarily concerns the relevance of the instrumental variable, which can be tested easily. Subsoil conditions will be valid instruments for city size in the identification of agglomeration effects as long as they are unrelated to the demand side of land markets. Notably, relevant subsoil conditions are not necessarily limited to bedrock. Curci (2017) proposes the interaction of earthquake risk and the share of buildings with an elevator as an instrument for building heights. He argues that earthquake risk disproportionately add to the cost of tall buildings. While Curci (2017) acknowledges that seismic risk is correlated with natural advantages (such as the presence of water) that can affect housing demand, he argues that the exclusion restriction will hold conditional on MSA fixed effects.

O.4 Potential for future research

Robust and systematic evidence on the effect of urbanization on building heights is surprisingly scarce. Moreover, one concern with the interpretation of correlations between city size and building height is that skyscrapers generate density and, therefore, can be cause and effect of urbanization. Future research could seek to establish a causal effect urbanization has had on skyscraper development using suitable instruments that are unrelated to the supply side of land markets (e.g. transport innovations). An important question to be addressed is why cities in the developed world tend to grow by building out and up wheres cities in the developing world grow by crowding in (Jedwab et al., 2020).

Likewise we still do not know much about the role tall buildings play in promoting urbanization. Given that construction innovations have radically reduced the cost of height, this is a notable research gap. Am ambitious goal in would be to quantify the contributions of the rise of agglomeration economies, transport improvements, and construction innovations to urbanization since the 19$^{\text{th}}$ century.

P Height regulation

This section complements Section 7 in the main paper. We briefly discuss forms and stringency of height regulations around the world, provide some background on the measurement of welfare in our open-city model, review the related research in greater detail and lay out potential for future research.

P.1 Forms of height regulation

When tall buildings started to rise in U.S. cities at the end of the 19$^{\text{th}}$ century, planners and officials were concerned about shadows, increased traffic congestion, increased risk of conflagrations, and correspondingly reduced property values of surrounding buildings (Heights of Buildings Commission, 1913; Hoxie, 1915). While some cities, like Chicago and Boston, placed direct height caps on their buildings, New York never did. In 1916, it implemented the first comprehensive zoning regulations in the nation. Rather than capping heights, it mandated setback rules that required tall buildings to set back from the street line as they rose taller. This gave rise to the so-called "wedding-cake style" architecture. The purpose of this regulation was to increase sunlight on the streets and reduce total building area and congestion. New York's codes were widely copied throughout the country in the 1920s (Weiss, 1992). These policies were not based on rigorous social cost-benefit analyses. Instead, planners followed their own normative judgements under the constraint that the imposed regulation should not be overturned by courts (Weiss, 1992; Bertaud, 2018).

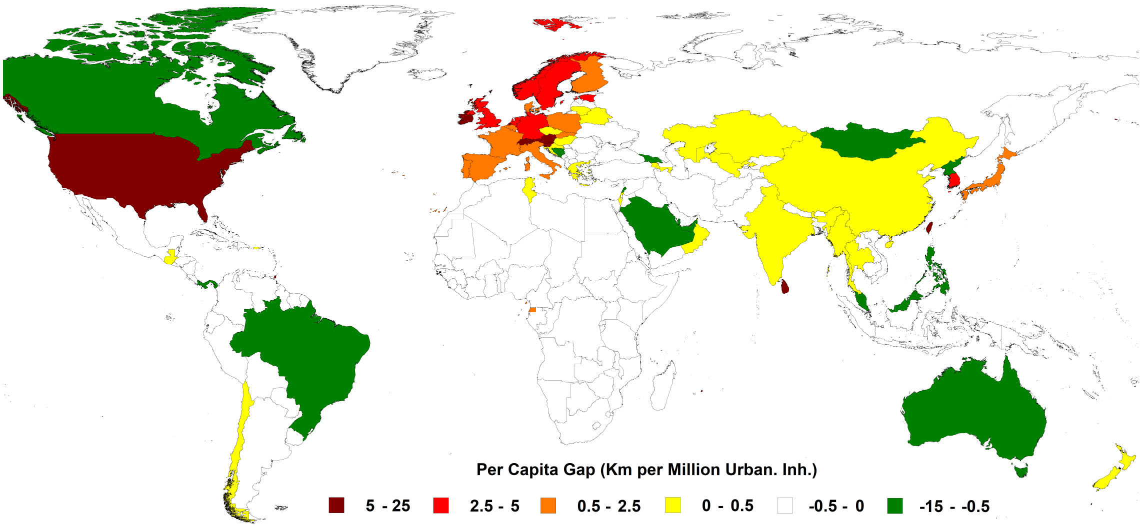
Today, cities around the world generally attempt to control building height and density by capping the maximum allowable floor area ratios (FARs). The FAR is the total building area divided by the lot area. For example, the maximum allowable FAR for Manhattan office buildings is 15 (or 18 if open space is provided). Similarly, for Chicago, the maximum FAR is 12. To put these numbers in perspective, the maximum FAR for Paris, a "flat" city, is 3 (Brueckner and Sridhar, 2012).

P.2 Stringency of height regulation

In keeping with intuition, Figure A13 shows a positive correlation between maximum allowable central-city FARs on the one hand, and the number and height of skyscrapers on the other, for cities around the world. The relationship, however, is quite noisy, suggesting that the causes and effects of regulation of vertical growth are highly context-dependent. As an example, the most vertical cities of the world, Hong Kong and New York have many and tall skyscrapers despite the FAR regulation being more restrictive than in Tokyo or Singapore.

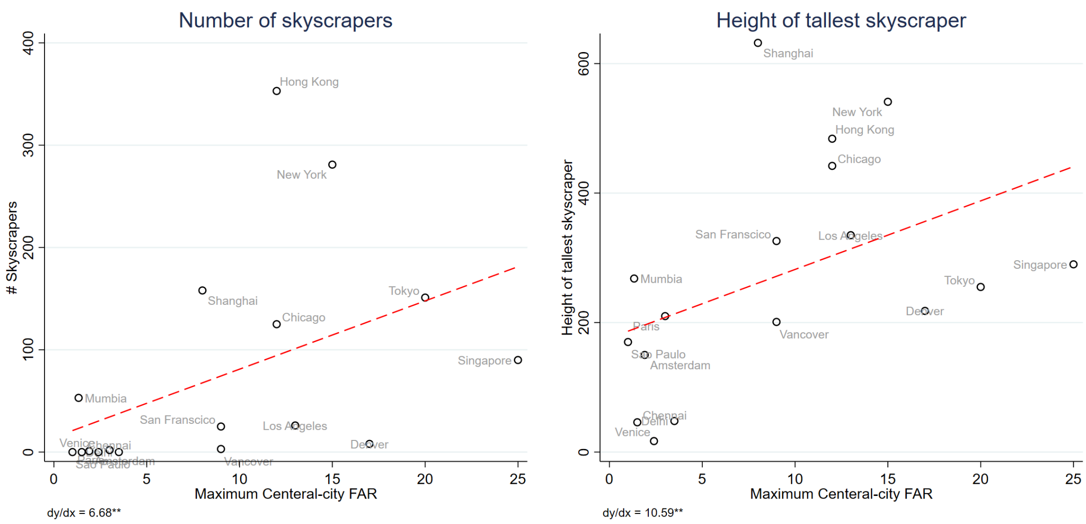
Using a database of nearly every 80-meter or taller building on the planet, Jedwab et al. (2020) compute the building-height gap by country. The measure, depicted in Figure A12, summarizes how much building height per capita falls short of the predicted value for given fundamentals under laissez-faire regulation. Considering their economic potential, European countries and the U.S. under-perform in terms of vertical growth, most likely due to planning regulations. The gap measure positively correlates with measures of sprawl, housing prices, and air pollution, suggesting costs of regulation.

Figure A12: Per-capita height gaps by country



Note: The height gap measures by how much total height of tall buildings in per-capita terms falls short of the predicted value under laissez-fair regulation (for given fundamentals). The measure is constructed for 149 countries in 2020. Positive values (red shades) indicate more stringent regulation. Source: Jedwab et al. (2020).

Figure A13: Skyscrapers and FAR regulation US



Sources: Floor area ratio data from Brueckner and Sridhar (2012). Skyscraper data: https://www.skyscrapercenter.com/

P.3 Welfare implications

Let's denote by $r(x) = \max(r^C(x), r^R(x), r^a(x))$ the equilibrium land rent at location x. Let's further denote by superscript * equilibrium outcomes in the unregulated scenario and by superscript ^{c} equilibrium outcomes under a heigt limit $\bar{S}^U$. The aggregate and rent in the unregulated city is

$$r^* = \int_{-x_1^*}^{x_1^*} r^*(x)dx$$

The counterfactual land rent over the same horizontal space in the presence of a height limit is

$$r^c = \int_{-x_1^*}^{x_1^*} r^c(x)dx,$$

so that the change in aggregate land rent within constant boundaries following the introduction of a height limit is given by

$$\Delta r = r^c - r^*.$$

Since by the nature of the Cobb-Douglas production function, output is $g = \frac{yN}{\alpha^C}$, we can write the change in land rent normalized by the aggregate land rent in the unregulated city as

$$\Delta r = \frac{\alpha^C \Delta r}{yN} g.$$

For a height limit of $\bar{S} = \bar{S}^C = \bar{S}^R = 10$, we find $\frac{\alpha^C \Delta r}{yN} = -5.8\%$ in Figure 17. Applying the 2012 GDP of Houston, TX of about \$400 billion, the change in aggregate land rent that would result from the height cap would amount to \$ 23 billion. Given a 2012 population of Houston of 2.1 million, this corresponds to about \$11 thousand per capita and year. At a 5%-capitalization rate, the effect on land value would exceed \$450 billion in total and \$210 thousand in per-capita terms.

P.4 Related literature

Gyourko and Molloy (2015) provide a review of a sizable literature on the causes and effects of effects of building regulation. The literature specifically concerned with height regulation is somewhat thinner but growing.

In general, it appears that FAR regulation tends to be binding. In several analyses, Barr (2010, 2012, 2013) concludes that zoning regulations in New York and Chicago reduce building heights. Ahlfeldt and McMillen (2018) find that in Chicago, the building footprints of taller buildings cover a smaller fraction of the land parcel. This relationship reflects that for developers to go tall, they must reduce the floor plate sizes in order to maintain the maximum allowable FAR. Sometimes, extra height is allowed in return for publicly accessible open space, further reducing the floor plate. Brueckner et al. (2017) show that the elasticity of land value with respect to FAR is a theory-consistent measure of the stringency of height regulation, which allows cities to be ranked in terms of stringency. Using this measure, Brueckner and Singh (2020) infer that New York and Washington D.C. have particularly stringent height regulation. Similarly, Moon (2019) infers that the stringency of regulation in Manhattan is the highest in New York City.

Naturally, height restrictions have welfare consequences, some of which are unintended. Bertaud and Brueckner (2005) and Brueckner and Sridhar (2012) demonstrate that the very low allowable FARs in Indian cities increase sprawl and traffic congestion. Hence, the attempt to reduce externalities from building density has produced a new suite of negative externalities. Since binding height restrictions, ceteris paribus, reduce housing supply, the general equilibrium effect is to raise house prices above the marginal cost of construction, adding to affordability problems (Glaeser et al., 2005).

In some cities, planners seek to mitigate negative externalities by ensuring that tall buildings are aesthetically appealing. Cheshire and Dericks (2020) study the impact of tight building height regulations in London, where skyscrapers have to go through an approval process since there is no "as-of-right" development as in New York or Chicago. The authors document that developers employing so-called Trophy Architects—those that won one of three prestigious architectural awards—are allowed to build 14-floors taller than otherwise, a sizable magnitude in a reasonably flat city. They interpret the one-and-a-half-fold increase in site value due to the extra permitted height as the compensation of the cost of rent-seeking and, thus, an indirect measure of deadweight loss.

Ironically, some of the unintended negative effects of height restrictions may have socially desirable second-order effects. Borck (2016) shows theoretically how building height regulation leads to lower housing consumption due to a supply-driven increase in house prices, which leads to a reduction in greenhouse gas emissions. In his model with a global emissions externality, the welfare effect is non-monotonic and, depending on the value of the externality, either the absence of height regulation or a very stringent regulation can be socially desirable.

P.5 Potential for future research

Much of the literature on height regulation is concerned with the negative collateral effects. One naturally wonders how these costs compare to potential benefits that motivate height regulations in the first instance. There is some evidence pointing to amenity values of sunshine (Fleming et al., 2018) and distinctive design (Ahlfeldt and Holman, 2018), suggesting that a regulation that reducse shadowing and improves the design of tall buildings may have positive effects. Also, while very tall buildings likely reduce sprawl and encourage mass transit, they tend to produce more CO_2 on per square meter, so that their environmental impact is theoretically ambiguous.

There is an evident polarization between opponents and proponents of tall buildings. The former argue that skyscrapers are, aesthetically speaking, "too big" for the "human scale" (Gehl, 2013) and an anathema to the vibrant city that Jane Jacobs (1961) argued for. The latter focus on the cost of height restrictions that materialize in sprawl and affordability problems. Yet, only a quantitative evaluation of the positive and negative external effects of tall buildings enables setting the appropriate Pigovian tax for a transition into a first-best optimum of building heights.

Likely, the degree of stringency of height regulation varies significantly across cities around the world because the external costs and benefits vary, too. Developing an understanding of external costs and benefits of skyscrapers that account of heterogeneity across institutional context is an ambitious research agenda that will likely remain topical for quite some time.

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